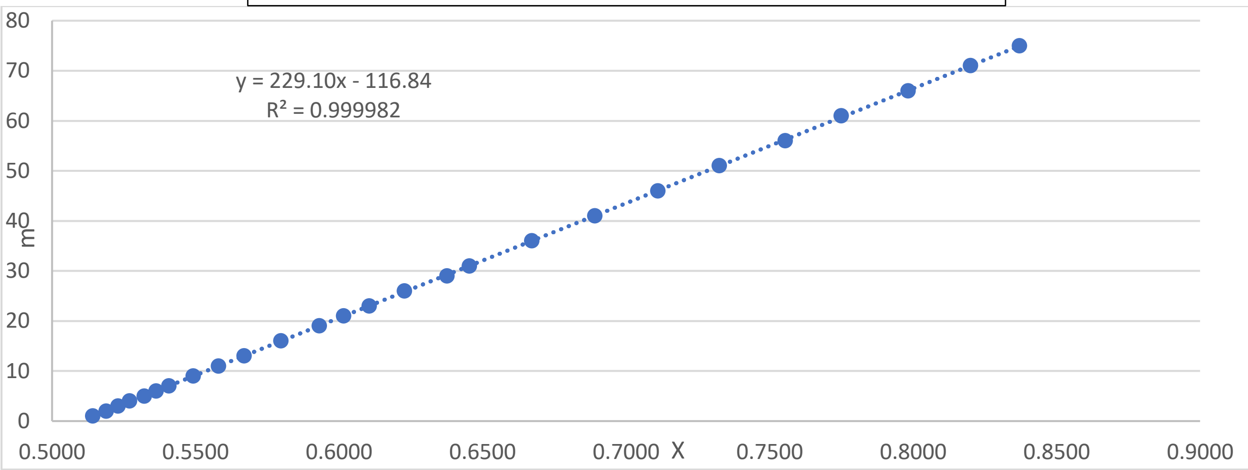
# Problem E2- Solution

| A.1 | | | |
|---|---|---|---|
| number | m | $\theta_m$ (degrees) | $\sqrt{n^2 - \sin^2\theta_m} - \cos\theta_m$ |
| 1 | 1 | 9.00 | 0.5142 |
| 2 | 2 | 13.00 | 0.5188 |
| 3 | 3 | 15.75 | 0.5229 |
| 4 | 4 | 18.00 | 0.5270 |
| 5 | 5 | 20.50 | 0.5322 |
| 6 | 6 | 22.25 | 0.5362 |
| 7 | 7 | 24.00 | 0.5406 |
| 8 | 9 | 27.00 | 0.5491 |
| 9 | 11 | 29.75 | 0.5579 |
| 10 | 13 | 32.25 | 0.5668 |
| 11 | 16 | 35.50 | 0.5798 |
| 12 | 19 | 38.50 | 0.5931 |
| 13 | 21 | 40.25 | 0.6015 |
| 14 | 23 | 42.00 | 0.6105 |
| 15 | 26 | 44.25 | 0.6228 |
| 16 | 29 | 46.75 | 0.6375 |
| 17 | 31 | 48.00 | 0.6453 |
| 18 | 36 | 51.25 | 0.6671 |
| 19 | 41 | 54.25 | 0.6891 |
| 20 | 46 | 57.00 | 0.7110 |
| 21 | 51 | 59.50 | 0.7325 |
| 22 | 56 | 62.00 | 0.7555 |
| 23 | 61 | 64.00 | 0.7750 |
| 24 | 66 | 66.25 | 0.7982 |
| 25 | 71 | 68.25 | 0.8200 |
| 26 | 75 | 69.75 | 0.8371 |

S2-2

A.2




**A.3**

B = 229.1

A = -116.8

**A.4**

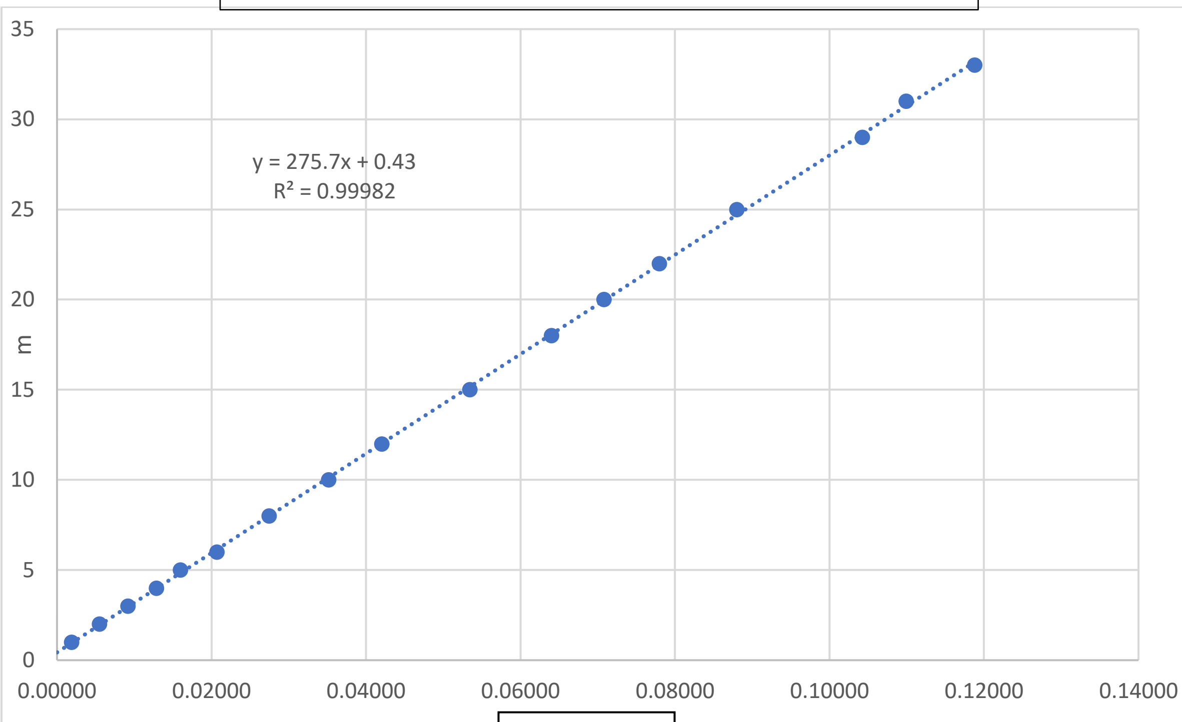
h = $(148.9 \pm 0.1)\ \mu m$

| B.1 | | | |
|---|---|---|---|
| number | m | $\theta_m$ (degrees) | ${\theta_m}^2$ (rad$^2$) |
| 1 | 1 | 2.50 | 0.00190 |
| 2 | 2 | 4.25 | 0.00550 |
| 3 | 3 | 5.50 | 0.00921 |
| 4 | 4 | 6.50 | 0.01287 |
| 5 | 5 | 7.25 | 0.01601 |
| 6 | 6 | 8.25 | 0.02073 |
| 7 | 8 | 9.50 | 0.02749 |
| 8 | 10 | 10.75 | 0.03520 |
| 9 | 12 | 11.75 | 0.04206 |
| 10 | 15 | 13.25 | 0.05348 |
| 11 | 18 | 14.50 | 0.06405 |
| 12 | 20 | 15.25 | 0.07084 |
| 13 | 22 | 16.00 | 0.07798 |
| 14 | 25 | 17.00 | 0.08803 |
| 15 | 29 | 18.50 | 0.10426 |
| 16 | 31 | 19.00 | 0.10997 |
| 17 | 33 | 19.75 | 0.11882 |

**B.2**

$$m = \frac{H}{2\lambda}\left(1-\frac{1}{n}\right){\theta_m}^2$$

B.3




$\theta_m^2\ (rad^2)$

**B.4**

B = $275.7$

A =$0.43$

**B.5**

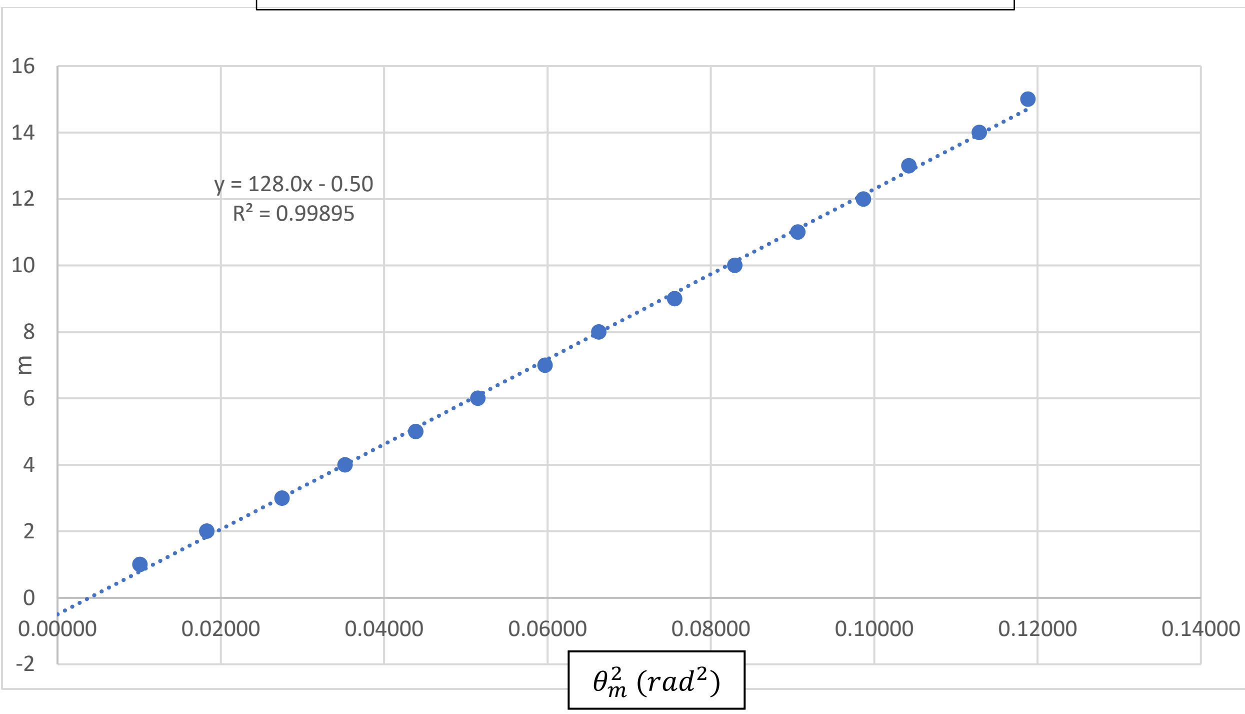
H = $(1.061 \pm 0.004)mm$

| C.1 | | | |
|---|---|---|---|
| number | m | $\theta_m$ (degree) | ${\theta_m}^2$ (rad$^2$) |
| 1 | 1 | 5.75 | 0.01007 |
| 2 | 2 | 7.75 | 0.01830 |
| 3 | 3 | 9.50 | 0.02749 |
| 4 | 4 | 10.75 | 0.03520 |
| 5 | 5 | 12.00 | 0.04386 |
| 6 | 6 | 13.00 | 0.05148 |
| 7 | 7 | 14.00 | 0.05971 |
| 8 | 8 | 14.75 | 0.06627 |
| 9 | 9 | 15.75 | 0.07556 |
| 10 | 10 | 16.50 | 0.08293 |
| 11 | 11 | 17.25 | 0.09064 |
| 12 | 12 | 18.00 | 0.09870 |
| 13 | 13 | 18.50 | 0.10426 |
| 14 | 14 | 19.25 | 0.11288 |
| 15 | 15 | 19.75 | 0.11882 |

**C.2**

$$m = \frac{H}{2\lambda}\left(N - \frac{N^2}{n}\right){\theta_m}^2$$

C.3




**C.4**

B = $128.0$

A = $-0.50$

**C.5**

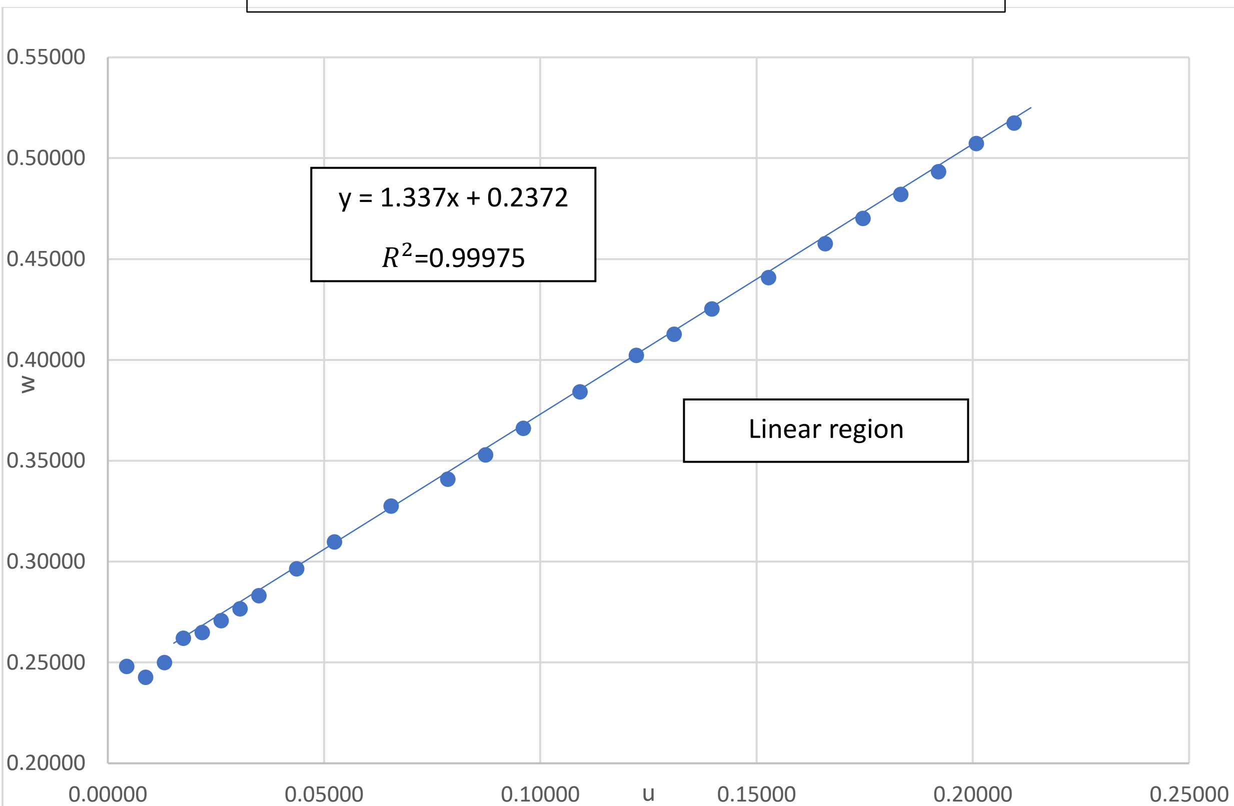
N =$1.332 \pm 0.002$

| D.1 | | | D.3 | |
|---|---|---|---|---|
| number | m | $\theta_m$ (degree) | u | w |
| 1 | 1 | 13.25 | 0.00436 | 0.24795 |
| 2 | 2 | 19.00 | 0.00873 | 0.24265 |
| 3 | 3 | 23.00 | 0.01309 | 0.24981 |
| 4 | 4 | 26.00 | 0.01746 | 0.26200 |
| 5 | 5 | 29.00 | 0.02182 | 0.26474 |
| 6 | 6 | 31.50 | 0.02619 | 0.27069 |
| 7 | 7 | 33.75 | 0.03055 | 0.27653 |
| 8 | 8 | 35.75 | 0.03492 | 0.28307 |
| 9 | 10 | 39.25 | 0.04365 | 0.29637 |
| 10 | 12 | 42.25 | 0.05238 | 0.30973 |
| 11 | 15 | 46.25 | 0.06547 | 0.32743 |
| 12 | 18 | 50.00 | 0.07857 | 0.34076 |
| 13 | 20 | 52.00 | 0.08730 | 0.35289 |
| 14 | 22 | 53.75 | 0.09603 | 0.36608 |
| 15 | 25 | 56.25 | 0.10912 | 0.38415 |
| 16 | 28 | 58.50 | 0.12222 | 0.40213 |
| 17 | 30 | 60.00 | 0.13095 | 0.41261 |
| 18 | 32 | 61.25 | 0.13968 | 0.42517 |
| 19 | 35 | 63.25 | 0.15277 | 0.44072 |
| 20 | 38 | 65.00 | 0.16587 | 0.45761 |
| 21 | 40 | 66.00 | 0.17460 | 0.47008 |
| 22 | 42 | 67.00 | 0.18333 | 0.48193 |
| 23 | 44 | 68.00 | 0.19205 | 0.49320 |
| 24 | 46 | 68.75 | 0.20078 | 0.50715 |
| 25 | 48 | 69.75 | 0.20951 | 0.51739 |

## D.2

$$u = \frac{m\lambda}{h} \qquad w = \frac{\frac{m\lambda}{h}\left(n + \frac{m\lambda}{2h}\right)}{1 - \cos\theta_m}$$

D.4




By removing the first 3 points

| D.5 |
|---|
| B = 1.337 |
| A = 0.2372 |

| D.6 |
|---|
| $N_B$ = 1.337±0.005 |
| $N_A$ = 1.3319 ± 0.0005 |

Theoretical calculations:

B.2 & C.2:

$$m = \frac{H}{\lambda}\left(\sqrt{n^2 - N^2 sin^2\theta_m} - Ncos\theta_m\right) - \frac{H}{\lambda}(n - N)$$

$$\theta_m \ll 1 : \qquad \sin\theta_m \approx \theta_m \quad ; \qquad \cos\theta_m \approx 1 - \frac{\theta_m^2}{2}$$

$$m = \frac{H}{\lambda}\left(n\sqrt{1 - \frac{N^2\theta_m^2}{n^2}} - N\left(1 - \frac{\theta_m^2}{2}\right)\right) - \frac{H}{\lambda}(n - N)$$

$$m = \frac{H}{\lambda}\left(n\left(1 - \frac{N^2\theta_m^2}{2n^2}\right) - N\left(1 - \frac{\theta_m^2}{2}\right)\right) - \frac{H}{\lambda}(n - N)$$

$$m = \frac{H}{2\lambda}N\left(1 - \frac{N}{n}\right)\theta_m^2$$

$$N = 1 : \; m = \frac{H}{2\lambda}\left(1 - \frac{1}{n}\right)\theta_m^2$$

D.2:

$$m = \frac{h}{\lambda}\left(\sqrt{n^2 - N^2 sin^2\theta_m} - Ncos\theta_m\right) - \frac{h}{\lambda}(n - N)$$

$$\frac{m\lambda}{h} + n - N(1 - cos\theta_m) = \sqrt{n^2 - N^2 sin^2\theta_m}$$

$$\left(\frac{m\lambda}{h}\right)^2 + 2n\left(\frac{m\lambda}{h}\right) + n^2 + N^2(1 - cos\theta_m)^2 - 2N(1 - cos\theta_m)\left(\frac{m\lambda}{h} + n\right) = n^2 - N^2 sin^2\theta_m$$

$$\left(\frac{m\lambda}{h}\right)^2 + 2n\left(\frac{m\lambda}{h}\right) + N^2(2 - 2cos\theta_m) - 2Nn(1 - cos\theta_m) - 2N(1 - cos\theta_m)\left(\frac{m\lambda}{h}\right) = 0$$

$$\frac{\frac{m\lambda}{h}\left(n + \frac{m\lambda}{2h}\right)}{1 - \cos\theta_m} + N^2 - Nn - N\left(\frac{m\lambda}{h}\right) = 0$$

$$\frac{\frac{m\lambda}{h}\left(n + \frac{m\lambda}{2h}\right)}{1 - \cos\theta_m} = N(n - N) + \left(\frac{m\lambda}{h}\right)N$$

$$w = N(n - N) + uN$$

Error calculations:

Linear equation slope and intercept uncertainties:

$$\Delta B = B\sqrt{\frac{1}{n-2}\left(\frac{1}{r^2}-1\right)} \qquad ; \qquad \Delta A = \Delta B\sqrt{\bar{x}^2+\sigma_x^2}$$

C.5:

$$B = \frac{H}{2\lambda}\left(N-\frac{N^2}{n}\right) \Rightarrow \left(N-\frac{N^2}{n}\right) = \frac{2\lambda}{H}B \equiv c$$

$$\Rightarrow \frac{\Delta c}{c} = \sqrt{\left(\frac{\Delta B}{B}\right)^2+\left(\frac{\Delta H}{H}\right)^2}$$

$$\left(N-\frac{N^2}{n}\right) = c \Rightarrow N = \frac{n}{2} \pm \sqrt{\left(\frac{n}{2}\right)^2 - c\,n}$$

A negative sign is unacceptable in this equation.

$$N = \frac{n}{2} + \sqrt{\left(\frac{n}{2}\right)^2 - c\,n} \Rightarrow \Delta N = \frac{n}{2\sqrt{\left(\frac{n}{2}\right)^2 - c\,n}}\Delta c$$

D.6:

$$N_A(n-N_A) = A \Rightarrow N_A = \frac{n}{2} \pm \sqrt{\left(\frac{n}{2}\right)^2 - A}$$

A negative sign is unacceptable in this equation.

$$N_A = \frac{n}{2} + \sqrt{\left(\frac{n}{2}\right)^2 - A} \Rightarrow \Delta N_A = \frac{\Delta A}{2\sqrt{\left(\frac{n}{2}\right)^2 - A}}$$

Of course, this is calculated by ignoring the error of u and w. Since there is an h value in u and w, the h error causes an error in this quantity that is not included.