Experiment

IPhO INTERNATIONAL PHYSICS OLYMPIAD 2023 TOKYO JAPAN

A1-1

English

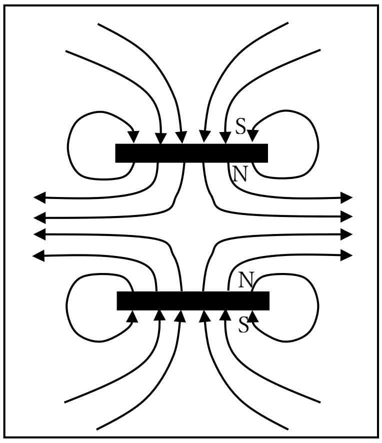
Mass Measurement (10 points)

Write down the numbers 0 to 9 in the following table:

0	1	2	3	4	5	6	7	8	9
0	1	2	3	4	5	6	7	8	9

Part A: Hooke's law and electromagnetic forces (2.4 points)

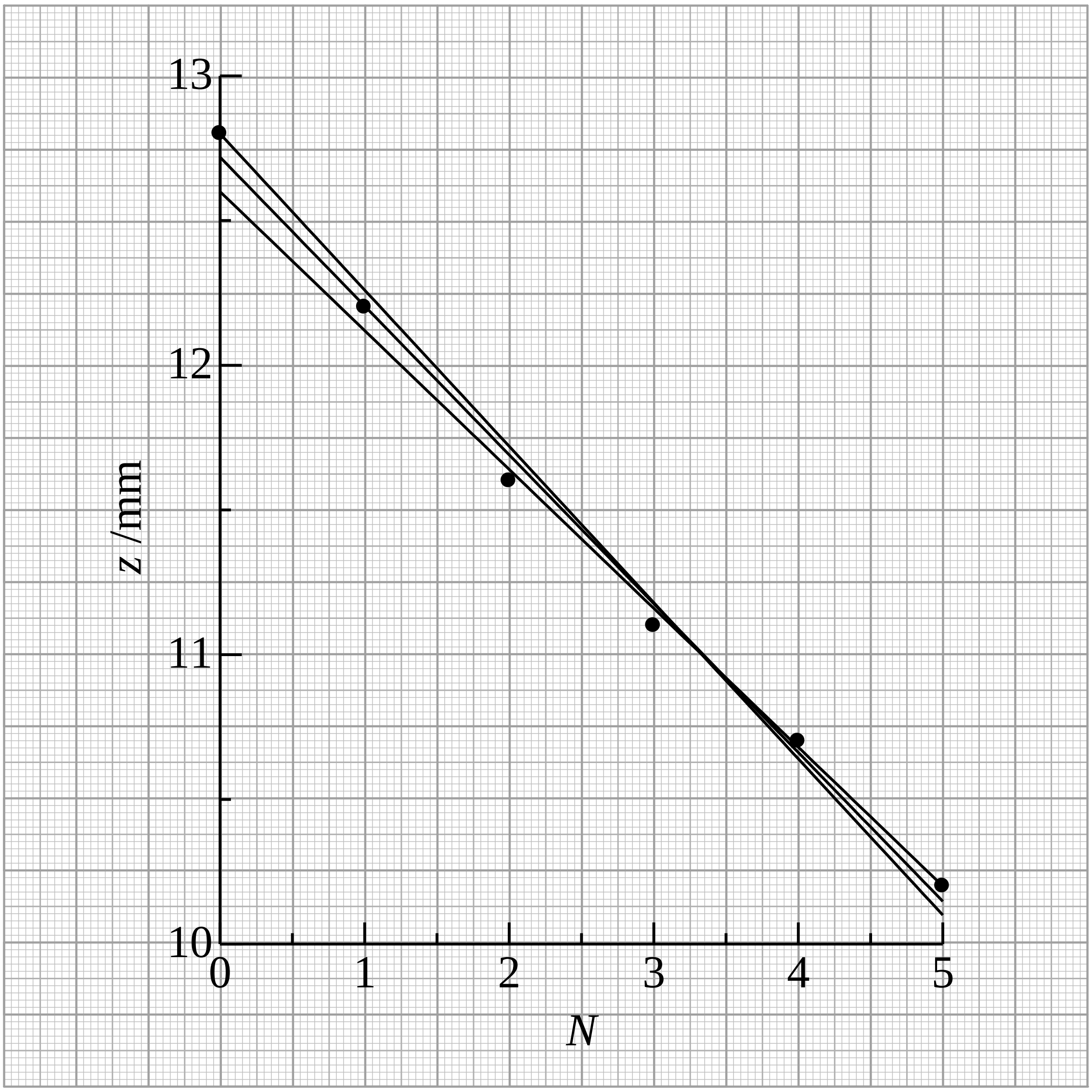
A.1 (0.4 pt)



A.2 (0.6 pt)

N	z /mm	I /A
0	12.8	0
1	12.2	0.103
2	11.6	0.213
3	11.1	0.323
4	10.7	0.423
5	10.2	0.524

A.3 (0.7 pt)



The slope and uncertainty are read from the lines plotted on the graph.

$$a = \frac{\Delta z}{\Delta N} = \frac{10.15 - 12.70}{5} = -0.51$$

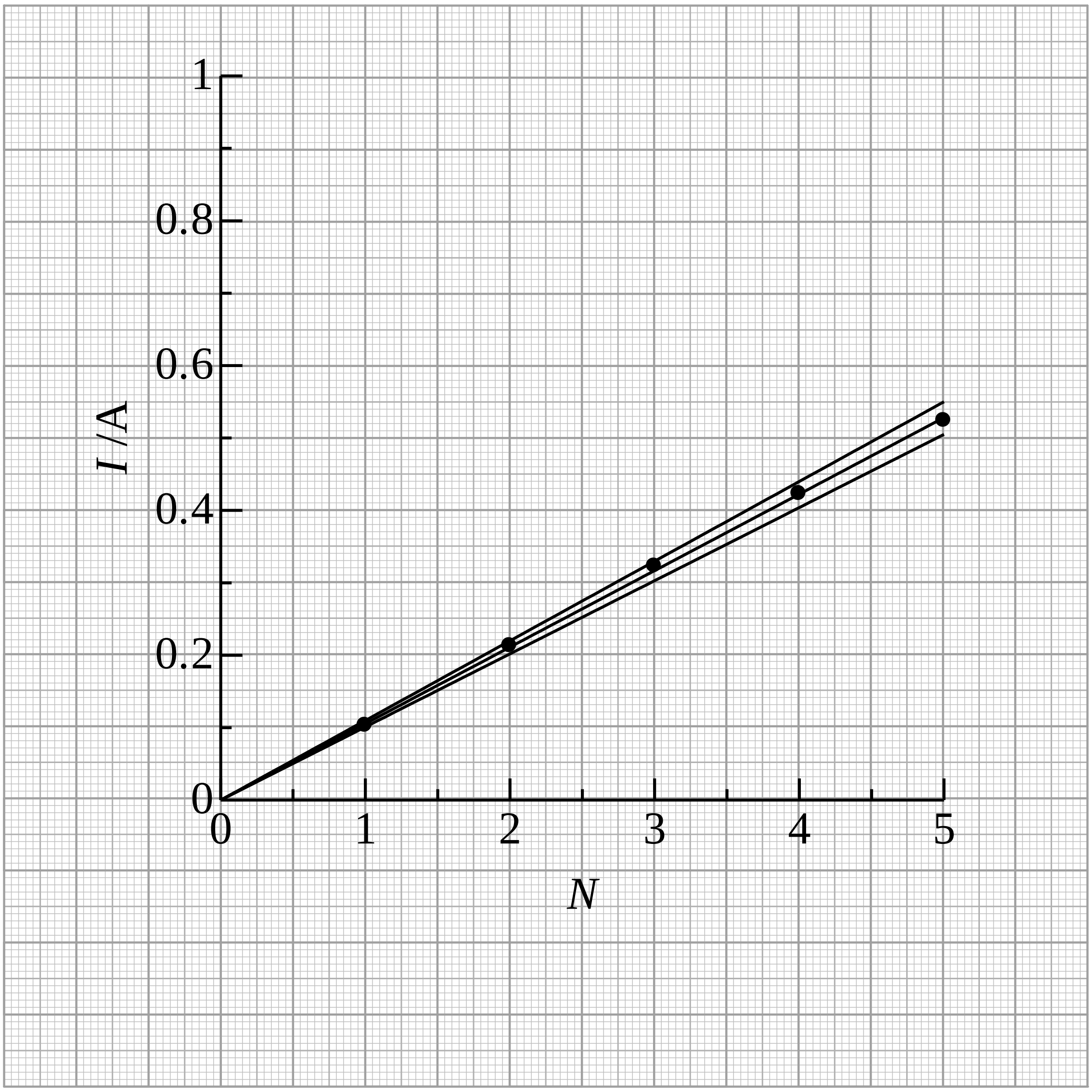
$$a_+ = \frac{10.20 - 12.60}{5} = -0.48$$

$$a_- = \frac{10.10 - 12.80}{5} = -0.54$$

$$\Delta a = \frac{-0.48 - (-0.54)}{2} = 0.03$$

$$a = \ -0.51 \pm 0.03 \text{ mm}$$

A.4 (0.7 pt)



The slope and uncertainty are read from the lines plotted on the graph.

$$b = \frac{I}{N} = \frac{0.53}{5} = 0.106$$

$$b_+ = \frac{0.55}{5} = 0.110$$

$$b_- = \frac{0.505}{5} = 0.101$$

$$\Delta b = \frac{0.110 - 0.101}{2} = 0.005$$

$$b = \ 0.106 \pm 0.005 \text{ A}$$

Part B: Induced electromotive force (3.0 points)

B.1 (0.2 pt)

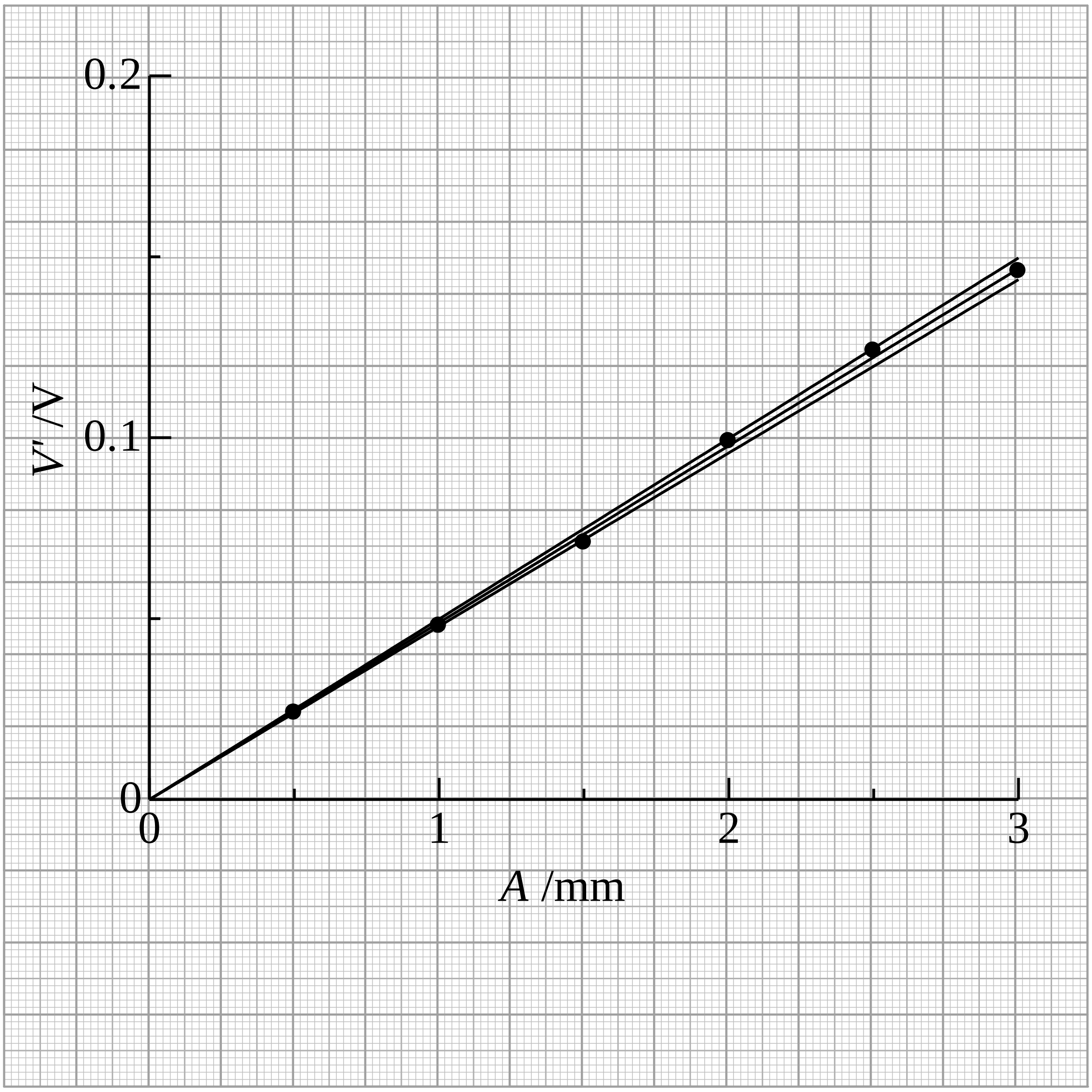
$V = 2\pi f A B L$

B.2 (0.5 pt)

$f_B = 15.85\ \mathrm{Hz}$

A /mm	V' /V
0.5	0.024
1.0	0.048
1.5	0.071
2.0	0.099
2.5	0.124
3.0	0.146

B.3 (0.7 pt)



The slope and uncertainty are read from the lines plotted on the graph.

$$c = \frac{V'}{N} = \frac{0.147}{5} = 0.049$$

$$c_+ = \frac{0.150}{5} = 0.050$$

$$c_- = \frac{0.144}{5} = 0.048$$

$$\Delta c = \frac{0.050 - 0.048}{2} = 0.001$$

$$c = \ 0.049 \pm 0.001 \text{ V/mm}$$

B.4 (0.4 pt)

$$BL = \frac{V}{2\pi A f_\mathrm{B}} = \frac{\sqrt{2}V'}{2\pi A f_\mathrm{B}} = \frac{\sqrt{2}c}{2\pi f_\mathrm{B}} = \frac{\sqrt{2}\times 0.049}{2\pi \times 15.85} = 0.000696\ \mathrm{Vs/mm} = 0.696\ \mathrm{Vs/m}$$

$$\Delta(BL) = BL \cdot \frac{\Delta c}{c} = 0.696 \times \frac{0.001}{0.049} = 0.014\ \mathrm{Vs/m}$$

$$BL = \ 0.696 \pm 0.014\ \mathrm{Vs/m}$$

B.5 (1.2 pt)

$$m = \frac{mg}{BL} \cdot \frac{BL}{g} = \frac{I}{N} \cdot \frac{V}{2\pi A f_\mathrm{B}} \cdot \frac{1}{g} = b\frac{\sqrt{2}c}{2\pi g f_\mathrm{B}} = 0.106 \times \frac{\sqrt{2}\times 0.049}{2\pi \times 9.80 \times 15.85} = 0.0075\ \mathrm{kg} = 7.5\ \mathrm{g}$$

The principle of the Kibble balance (watt balance)

Mechanical power: $Fv = Nmg \cdot 2\pi A f_\mathrm{B}$

Electrical power: VI

$Fv = VI$

$$\Delta m = m \cdot \sqrt{(\frac{\Delta b}{b})^2 + (\frac{\Delta c}{c})^2} = 0.4\ \mathrm{g}$$

$$m = \ 7.5 \pm 0.4\ \mathrm{g}$$

$$k = -\frac{mg}{a} = -\frac{7.5 \times 9.80}{-0.51} = 144\ \mathrm{N/m}$$

$$\Delta k = k \cdot \sqrt{(\frac{\Delta a}{a})^2 + (\frac{\Delta m}{m})^2} = 11\ \mathrm{N/m}$$

$$k = \ 144 \pm 11\ \mathrm{N/m}$$

Part C: Mass dependence of resonant frequency (2.3 points)

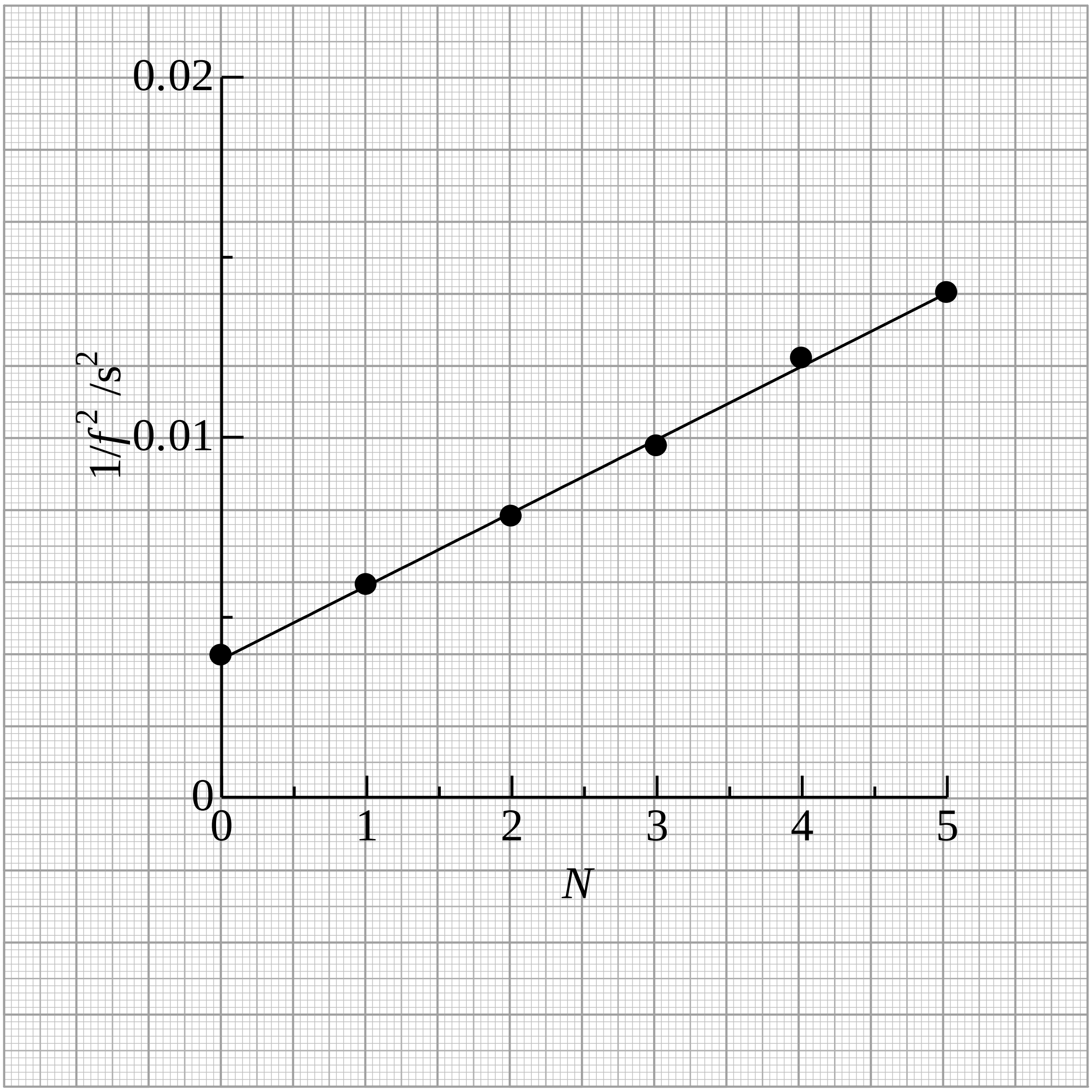
C.1 (0.2 pt)

$$f = \frac{1}{2\pi}\sqrt{\frac{k'}{M + Nm}}$$

C.2 (0.5 pt)

N	f /Hz	$1/f^2$ /s^2		
0	15.96	0.003926		
1	13.03	0.005390		
2	11.33	0.007790		
3	10.13	0.009745		
4	9.06	0.01218		
5	8.45	0.01401		

C.3 (1.0 pt)



The additional quantities $1/f^2$ are calculated in Table C.2. Then, $\frac{M}{k'}$ and $\frac{m}{k'}$ are obtained from the graph using the equation $\frac{1}{f^2} = (2\pi)^2(\frac{M}{k'} + \frac{m}{k'}N)$.

$$\frac{M}{k'} = \frac{0.0039}{(2\pi)^2} = 9.88 \times 10^{-5}\ \mathrm{s}^2$$

$$\frac{m}{k'} = \frac{(0.0140 - 0.0039)/5}{(2\pi)^2} = 5.12 \times 10^{-5}\ \mathrm{s}^2$$

C.4 (0.6 pt)

$$\frac{M}{m} = \frac{M/k'}{m/k'} = \frac{9.88}{5.12} = 1.93$$

$$\frac{M}{m} = 1.93$$

$$M = \frac{M}{m} \cdot m = 1.93 \times 0.0075 = 0.0145 \text{ kg}$$

$$M = 14.5 \text{ g}$$

$$k' = \frac{M}{M/k'} = \frac{0.0145}{9.88 \times 10^{-5}} = 147 \text{ N/m}$$

$$k' = 147 \text{ N/m}$$

Part D: Resonance characteristics (2.3 points)

D.1 (0.4 pt)

$V'_{\mathrm{AC}} = 0.157\ \mathrm{V}$

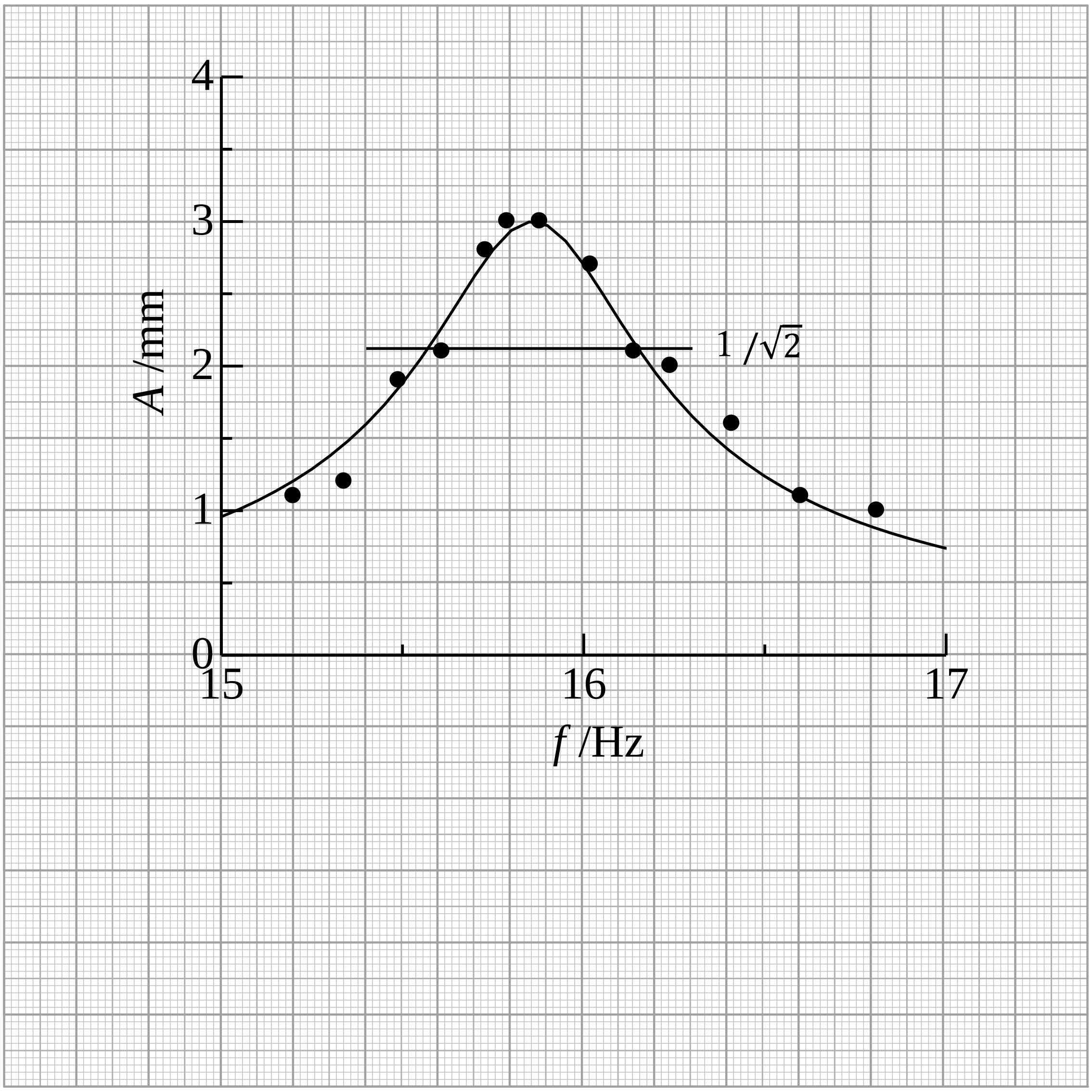
$F_{\mathrm{AC}} = BLI_{\mathrm{AC}} = BL \times 0.106 \times \sqrt{2}V'_{\mathrm{AC}} = 0.696 \times 0.106 \times \sqrt{2} \times 0.157 = 0.0164\ \mathrm{N}$

$F_{\mathrm{AC}} = 0.0164\ \mathrm{N}$

D.2 (0.9 pt)

f /Hz	A /mm	$(f - f_0)^2$ /Hz^2	$1/A^2$ /mm^{-2}
15.88	3.0	0.0064	0.111
15.79	3.0	0.0289	0.111
15.73	2.8	0.0529	0.128
15.61	2.1	0.1225	0.227
15.49	1.9	0.2209	0.277
15.34	1.2	0.3844	0.694
15.20	1.1	0.5776	0.826
16.02	2.7	0.0036	0.137
16.14	2.1	0.0324	0.227
16.24	2.0	0.0784	0.250
16.41	1.6	0.2025	0.391
16.60	1.1	0.4096	0.826
16.81	1.0	0.7225	1.000

D.2 (cont.)



D.3 (1.0 pt)

Reading from the graph D.2

$f_0 = 15.83$ Hz

$A(f_0) = 3.0$ mm

$$\Delta f = \frac{15.14 - 15.56}{2} = 0.29 \text{ Hz}$$

Calculaton using Eq.(4)

$$M = \frac{F_{AC}}{8\pi^2 f_0 \Delta f A(f_0)} = \frac{0.0164}{8\pi^2 \times 15.83 \times 0.29 \times 0.003} = 0.0151 \text{ kg}$$

$$M = 15.1 \text{ g}$$

An alternative way to find M

$(f - f_0)^2$ and $1/A^2$ are calculated in Table D.2 to use the linear relationship

$$\frac{1}{A^2} = (\frac{8\pi^2 M f_0}{F_{AC}})^2 \cdot [(f - f_0)^2 + (\Delta f)^2] \,.$$

$f_0 = 15.96$ Hz obtained in C.2 is used.

The slope is obtained from the graph of the additional quantities or the calculation

$$(\frac{8\pi^2 M f_0}{F_{AC}})^2 = 1.31 \text{ mm}^{-2}\text{Hz}^{-2} = 1.31 \times 10^6 \text{ m}^{-2}\text{Hz}^{-2}.$$

$$M = \sqrt{1.31 \times 10^6} \times \frac{F_{AC}}{8\pi^2 f_0} = \sqrt{1.31 \times 10^6} \times \frac{0.0164}{8\pi^2 \times 15.96} = 0.0149 \text{ kg}$$

$$M = 14.9 \text{ g}$$