Thickness Measurements Using Birefringence (10 points)

Uncertainty analysis is not required throughout this question.

Birefringence is an optical property of a crystal that light propagates as two rays experiencing different refractive indices. When the orthogonal crystal axes x and y lie in the plane of the input face of a birefringent crystal (Fig. 1), the electric field $\boldsymbol{E}$ of linearly polarized light at normal incidence on the crystal is decomposed into two orthogonal components $\boldsymbol{E}_x$ and $\boldsymbol{E}_y$ accompanied by refractive indices n_o and n_e, respectively. For a crystal of thickness L, the phase shift of the x-polarized light Γ_x and that of the y-polarized light Γ_y as they pass through the crystal are respectively given by

$$\Gamma_x = \frac{2\pi}{\lambda} n_\text{o} L, \tag{1}$$

$$\Gamma_y = \frac{2\pi}{\lambda} n_\text{e} L, \tag{2}$$

where λ is the wavelength of light in vacuum.

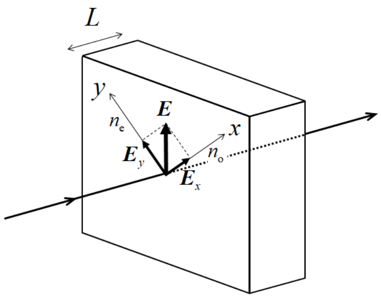


Figure 1: Vectorial decomposition of the electric field $\boldsymbol{E}$ of linearly polarized light at normal incidence on the surface of a birefringent crystal.

The phase difference Γ between the two rays is

$$\Gamma = \Gamma_y - \Gamma_x = \frac{2\pi}{\lambda} \Delta n L, \tag{3}$$

where

$$\Delta n = n_\text{e} - n_\text{o} \tag{4}$$

is the birefringence. Since the electric field of light is the vectorial sum of $\boldsymbol{E}_x$ and $\boldsymbol{E}_y$ with a phase difference Γ, the light after passing through the crystal has a polarization component perpendicular to the initial linear polarization of the incident light.

Let $I_\parallel$ and $I_\perp$ denote the intensities of the components of the light after passing through the crystal which are parallel and perpendicular to the direction of the linear polarization of the incident light, respectively. Hereafter the direction of the linear polarization of the incident light ($\boldsymbol{E}$ in Fig. 1) is 45° with respect to the x axis. Then the normalized intensity of the perpendicular component I_Norm is given by

$$I_\text{Norm} = \frac{I_\perp}{I_\text{Total}} = \sin^2 \frac{\Gamma}{2}, \tag{5}$$

where I_{Total} is the total transmitted light intensity, $I_{\parallel} + I_{\perp}$.

We can design an experiment such that I_{Norm} oscillates between 0 and 1 as we vary the wavelength of the incident light. Let λ_m $(m = 1, 2, 3, \cdots)$ be the wavelengths at which $I_{\text{Norm}} = 0$; then we find the phase difference Γ_m such that

$$\Gamma_m = \frac{2\pi}{\lambda_m} \Delta n(\lambda_m) L = 2\pi m. \tag{6}$$

This equation allows us to determine the crystal thickness L if multiple λ_m's can be measured for the known $\Delta n(\lambda_m)$.

In this experiment, you will determine the thickness of the quartz plate. Quartz is birefringent with its refractive indices n_{o} and n_{e} depending on the wavelength of light in vacuum as shown in Fig. 2.

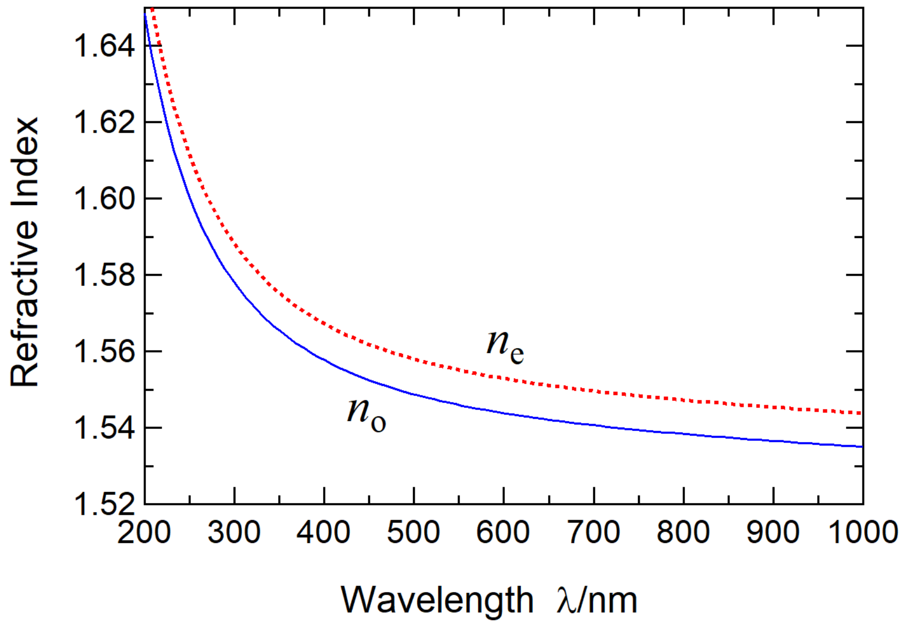
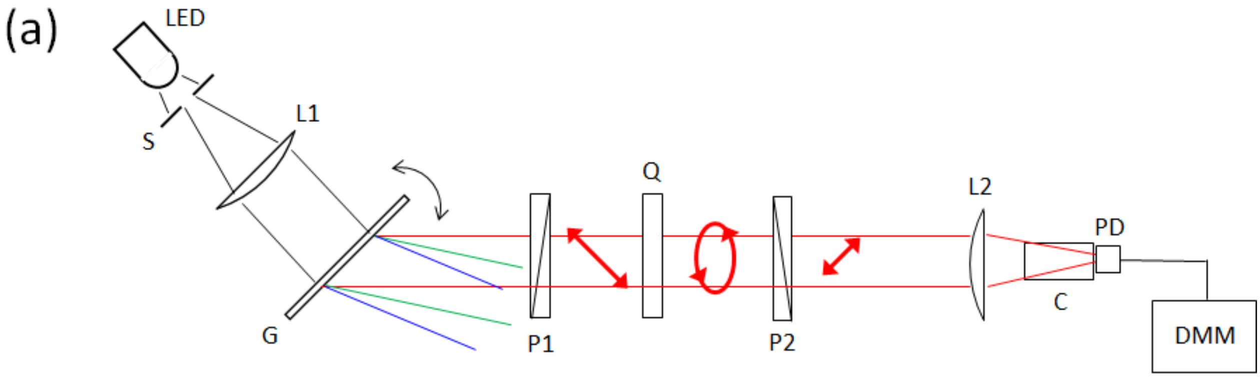


Figure 2: Wavelength dependence of the refractive indices n_{o} and n_{e} of quartz.

Figure 3 shows the thickness-measurement system. Shown in Figs. 4 and 5 are the optomechanical and photonic components and devices. A white light-emitting diode (LED) is used as the light source, which contains a blue LED and a phosphor. When light from the blue LED is irradiated onto the phosphor, white light is emitted with a continuous spectrum. Light from this white LED is dispersed, i.e., spectrally resolved, using the transmission diffraction grating **G**, and linearly polarized by the polarizer **P1**. Its direction of polarization ($\boldsymbol{E}$ in Fig. 1) is 45° off the x-axis of the quartz plate **Q**. The polarization component of light after passing through **Q**, i.e., parallel and perpendicular to the direction of polarization of **P1**, is selected by rotating the polarizer **P2**. The photodetector measures the light intensity.



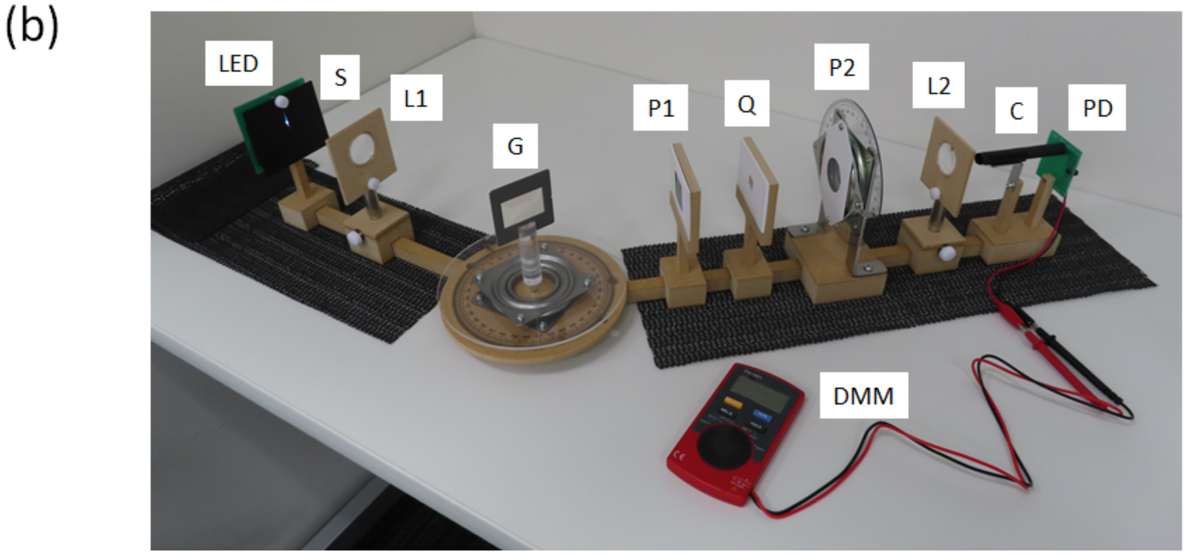


Figure 3: (a) Schematic and (b) photograph of thickness−measurement system. **LED**: white LED, **S**: slit, **L1**: collimating lens, **G**: transmission diffraction grating, **P1**: polarizer, **Q**: quartz plate, **P2**: polarizer, **L2**: focusing lens, **C**: light-shield cylinder, **PD**: photodetector, **DMM**: digital multimeter.

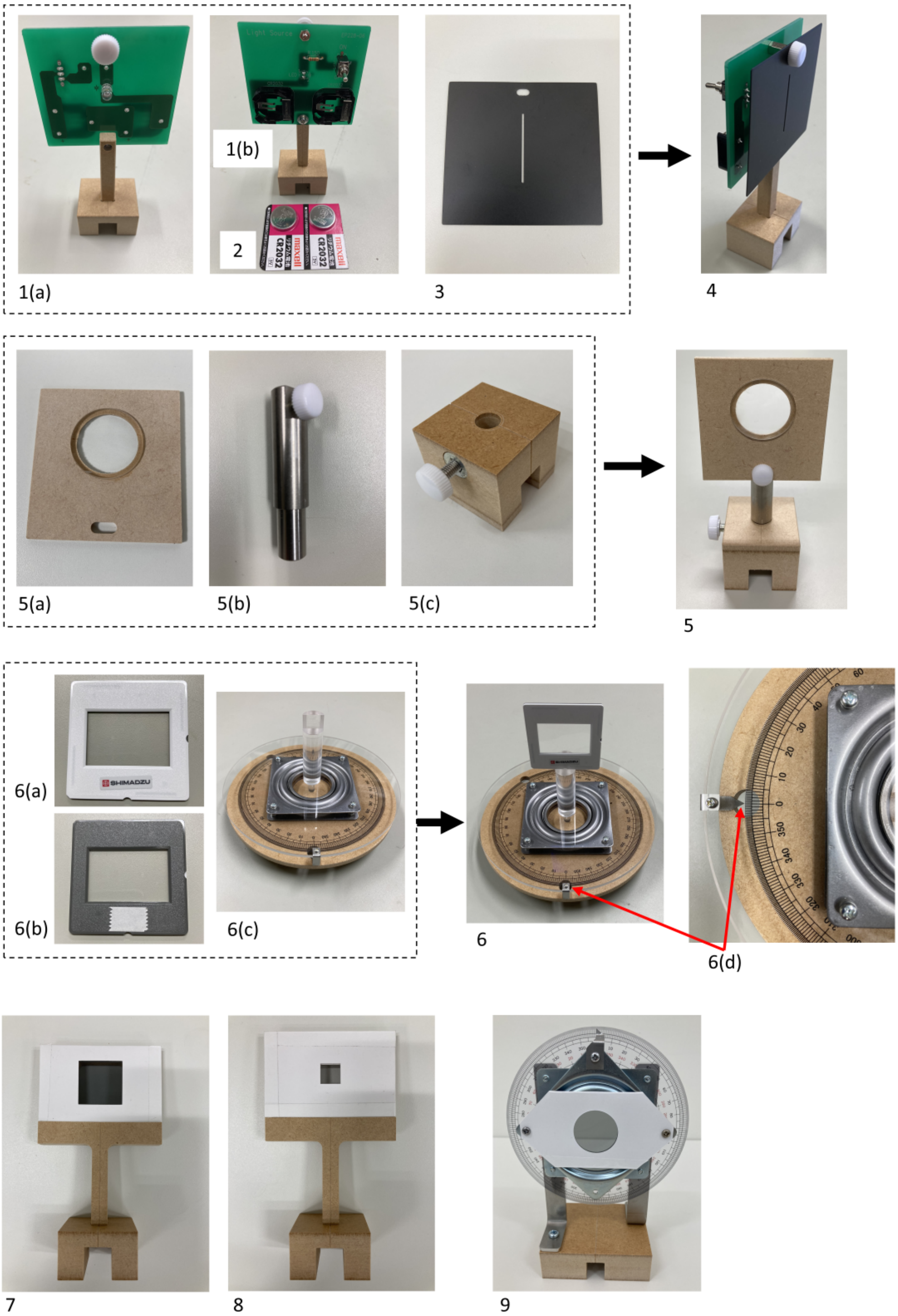


Figure 4: Components and devices: **1(a)**. white LED (front view); **1(b)**. white LED (rear view); **2**. batteries; **3**. slit (**S** in Fig. 3); **4**. LED with slit attached; **5**. lens (**L1**, **L2** in Fig. 3); **5(a)** mounted lens; **5(b)** lens post; **5(c)** post base; **6**. transmission diffraction grating (**6(a)** front; **6(b)** rear w/ adhesive tape) on **6(c)** rotation stage (**G** in Fig. 3); **6(d)** angle readout device on the rotation stage; **7**. polarizer (**P1** in Fig. 3); **8**. quartz plate (**Q** in Fig. 3); **9**. polarizer on rotation mount (**P2** in Fig. 3).

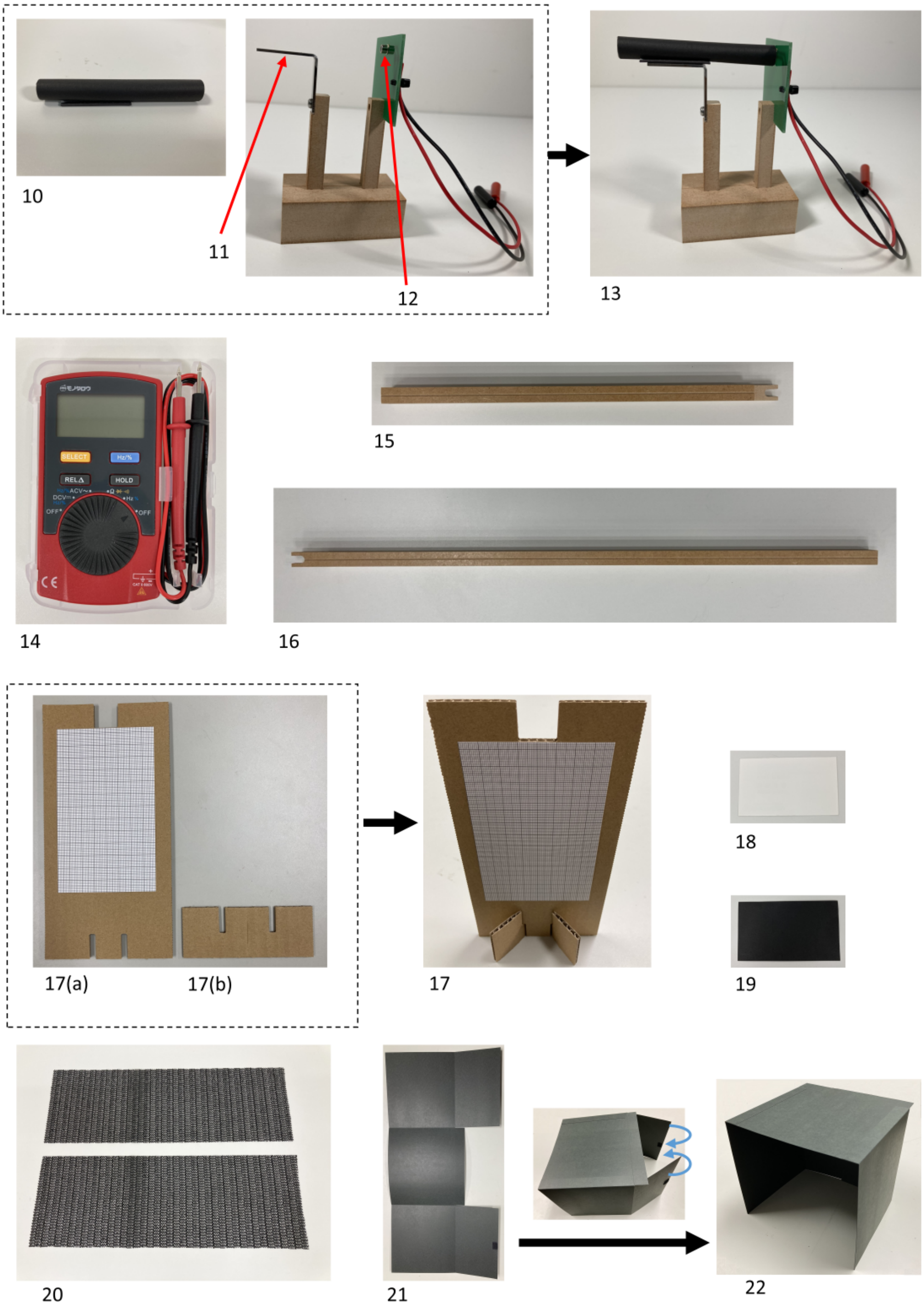


Figure 5: Components and devices (continued): **10**. light–shield cylinder with magnet (**C** in Fig. 3); **11**. cylinder mount; **12**. photodetector (**PD** in Fig. 3); **13**. photodetector with cylinder; **14**. digital multimeter (**DMM** in Fig. 3); **15**. short guide rail; **16**. long guide rail; **17**. scale assembly; **18**. white card; **19**. black card; **20**. anti-slip sheets; **21** & **22**. light-shield box (before assembly and as assembled).

Part A. Measurement System Setup (2.3 points)

The LED output is incident on the grating surface (Fig. 6). The rotation angle θ of **G** for normal incidence is defined as 0°. The counterclockwise and clockwise rotations are denoted by $+$ and $-$, respectively. The first-order diffraction angle α is defined as illustrated. Using the groove period (or slit separation) d of **G**, the wavelength λ is given in terms of θ as

$$\lambda = d\sin(\alpha - \theta) + d\sin\theta \tag{7}$$

$$= 2d\sin\frac{\alpha}{2}\cos\left(\frac{\alpha}{2} - \theta\right). \tag{8}$$

Hereafter use $d = 1.00$ μm and the fixed diffraction angle $\alpha = 40.0°$.

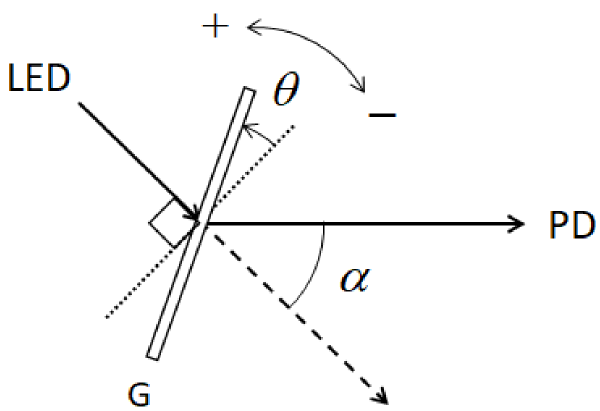


Figure 6: The rotation angle θ of the transmission diffraction grating **G** and the diffraction angle α.

A.1	Calculate the longest wavelength λ that can be measured and the associated θ.	0.3pt

A.2	Calculate the numeric values of θ for $\lambda = 440$ nm.	0.2pt

Setup procedures for the measurement system are as follows.

[1] Stand the scale assembly upright (**17** in Fig. 5) using the pedestal (**17(b)**).

[2] Set two batteries on the white LED module. The "+" sides must face toward you.

[3] Turn on the LED.

[4] Remove the screw on the front side of the LED module. Attach the slit to the LED module with the screw (**4** in Fig. 4). Using the scale assembly, adjust the slit position to make the transmitted white light flux brightest, and measure the height of the beam center at the exit of the slit (for the procedure [9]).

[5] Let the U-shaped open-slotted end of the long guide rail ride on that of the short one (Fig. 7(i)). Insert the rotation axle sticking out of the bottom face of the rotation stage into the "virtual through-hole" made by the guide rails (Fig. 7(ii)). Ensure free and smooth rotation of both arms about the axle referring to Fig. 7(iii). Make sure that the long guide rail will stay on the table $0° \leq \alpha \leq 40.0°$.

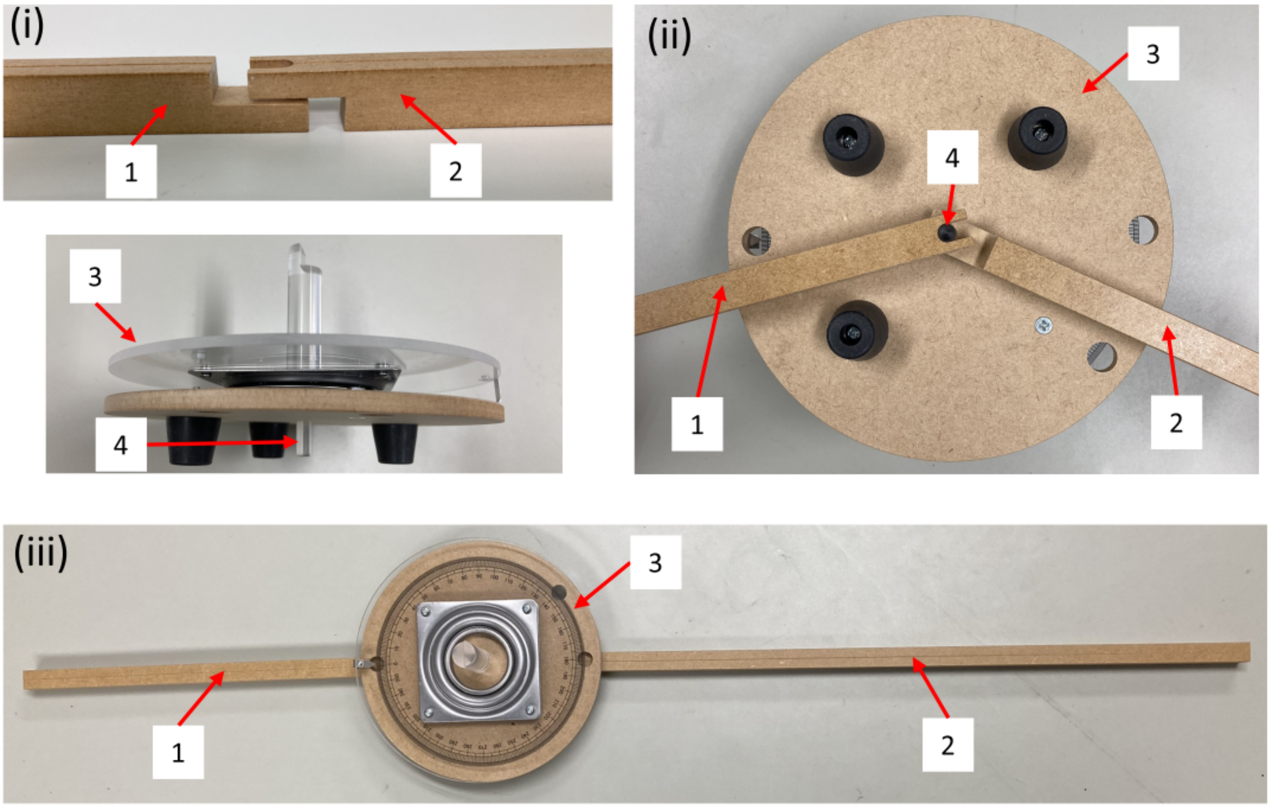


Figure 7: **(i)** U-shaped open-slotted end of the short guide rail under that of the long guide rail making a "virtual" through-hole. **(ii)** Into the virtual hole, insert the axle sticking out of the bottom face of the rotation stage. **(iii)** Top view of the rotation stage with guide rails that are free to rotate about the axle. **1**. short guide rail; **2**. long guide rail; **3**. rotation stage; **4**. axle of the rotation stage.

[6] Align the centerline of the short guide rail with 0° on the scale of the rotation stage, and keep it in that place. You may put an anti-slip sheet under the short guide rail.

[7] Assemble the lenses (**5** in Fig. 4).

[8] Place the white LED module with the slit and the lens (**L1** in Fig. 3) on the short guide rail. Adjust the distance between the slit and **L1** so that the light beam size after passing through **L1** remains almost constant, i.e., collimated, over the flight path.

[9] Using the scale assembly, measure the beam height after **L1**. Adjust the level of **L1** by loosening the setscrew of the post base and moving the post as necessary to keep the beam height almost the same as that right after the slit.

[10] Align the centerline of the long guide rail with 180° on the angle scale on the rotation stage.

[11] Tweak the horizontal position of the lens mount (**5(a)** in Fig. 4) by loosening the setscrew and moving it right or left. The beam center after **L1** should align with the center line of the long guide rail. You may put the scale assembly upside down over the long rail.

[12] Expose the second surface of the double-sided adhesive tape on the rear side of the transmission diffraction grating (**6(b)** in Fig. 4) and affix it to the axle top of the rotation stage (**6** in Fig. 4).

[13] Face the front side of the grating towards the light source, and rotate the stage so that the reflected light enters the slit, i.e., $\theta = 0°$ (normal incidence). Record the angle θ_{Stage} of the rotation stage. It will be used in B.1.

[14] Move the long guide rail around the axle so that $\alpha = 40.0°$ (Fig. 6). Once fixed, you may place another anti-slip sheet thereafter to prevent accidental misalignment.

[15] Place the lens (**L2** in Fig. 3) and the photodetector (**PD** in Fig. 3) with the cylinder mount on the long rail. To focus the diffracted light onto **PD**, adjust the distance between **PD** and **L2** along the long rail, and also the height of **L2**. The vertical beam diameter is thereby minimized. Check the beam diameter with the white card. In case it is too weak to recognize with the naked eye, use the light-shield box to cover **PD**.

[16] Set the light-shield cylinder to the mount (**13** in Fig. 5). The light shield minimizes the unwanted light to be detected.

[17] Connect **PD** to the DMM. The red (black) jump wire goes to red (black) terminal. Set the multimeter to the DC voltage measurement mode.

[18] Adjust the height of **L2** to maximize the DMM readings. Hereafter the intensity of light is identified with the voltage values on the DMM.

A.3	Rotate the rotation stage and find the angle θ and the corresponding wavelength λ_{Peak} at which the blue LED spectral density is maximized, assuming that $\alpha = 40.0°$. If your answer for λ_{Peak} is between 450 and 460 nm, your apparatus is properly aligned; write down $\alpha = 40.0°$ on the answer sheet and continue. Otherwise, you will have to find the true value of α. Without changing anything, including your original value for λ_{Peak}, find a corrected value for α which would make λ_{Peak} fall in the appropriate range. Record this α on the answer sheet and use it for the rest of the problem.	0.8pt

[19] Set the polarizers (**P1** and **P2** in Fig. 3) on the long guide rail.

A.4	Set the rotation stage to the $\theta = -15.0°$ position. Watch the readings on the DMM and find the angle $\varphi_\perp$ of the rotation mount of the polarizer **P2** such that its polarization direction is perpendicular to that of the light transmitted through the polarizer **P1**. From this result, find the angle $\varphi_\parallel$ of the rotation mount of the polarizer **P2** when its polarization direction is parallel to that of the polarizer **P1**.	0.3pt

A.5	Block the light through the slit by placing the black card in front of the slit. By doing so, you can evaluate the system background, i.e., the offset of the intensity from zero. We define the light intensities $I_{\text{Offset}\,\perp}$ and $I_{\text{Offset}\,\parallel}$ when the angles of the rotation mount of the polarizer **P2** are $\varphi_\perp$ and $\varphi_\parallel$, respectively. Measure the offsets $I_{\text{Offset}\,\perp}$ and $I_{\text{Offset}\,\parallel}$. Note that $I_{\text{Offset}\,\perp}$ and $I_{\text{Offset}\,\parallel}$ are due to light other than the light source. They should be eliminated by subtraction to determine the true contribution from the light source.	0.2pt

A.6	$I_\perp$ and $I_\parallel$ refer to the light intensities from the light source when the angles of the rotation mount of the polarizer **P2** are $\varphi_\perp$ and $\varphi_\parallel$, respectively. Measure the light intensities $I_\perp$ and $I_\parallel$ for $\theta = -15.0°$.	0.5pt

Part B. Measurement of transmitted light intensities (4.7 points)

Hereafter use the values of λ calculated using the corrected value of α in **A.3** as necessary.

B.1	Place the quartz plate between polarizers **P1** and **P2** and measure the transmitted light intensities $I_\perp$ and $I_\parallel$ at various angles θ. Your measurements should fully cover the wavelength range of 440 nm to 660 nm. Tabulate the following parameters: θ_{Stage} (angle readings of the rotation stage), $\theta,\ \lambda,\ I_\perp,\ I_\parallel,\ I_{\text{Total}} = I_\perp + I_\parallel,\ I_{\text{Norm}} = I_\perp / I_{\text{Total}}$. Note that when the value of θ_{Stage} increases, the value of θ decreases with the same value, and vice versa. You do not have to use every row of the provided table, but you should take enough data to obtain accurate results.	2.0pt

B.2	Plot the spectrum of the white LED, i.e., I_{Total}, versus wavelength on the graph.	1.0pt

B.3	Find the full width at half maximum $\Delta\lambda_{\text{FWHM}}$ of the spectrum of the blue LED built in the white LED. It is the width of a peak measured between those points which are at half the maximum amplitude	0.2pt

B.4	Plot the spectrum of I_{Norm} on the graph.	1.5pt

Part C. Analyses of Measured Results (3.0 points)

C.1	From the I_{Norm} graph, find all the wavelengths at which the intensities go through local minima. The associated order number m according to Eq. (6) must be given below the corresponding wavelength. To determine the birefringence Δn, use the values of n_{o} and n_{e} given in Table 1.	1.5pt

C.2	Obtain the sample thickness L.	1.5pt

Table 1: Refractive indices n_o and n_e of quartz (400–700 nm).

λ/nm	n_o	n_e	λ/nm	n_o	n_e	λ/nm	n_o	n_e
400	1.55769	1.56725	434	1.55394	1.56337	467	1.55107	1.56041
401	1.55756	1.56712	435	1.55384	1.56327	468	1.55099	1.56033
402	1.55744	1.56700	436	1.55374	1.56318	469	1.55091	1.56025
403	1.55732	1.56687	437	1.55365	1.56308	470	1.55084	1.56017
404	1.55720	1.56674	438	1.55355	1.56298	471	1.55076	1.56009
405	1.55707	1.56662	439	1.55346	1.56288	472	1.55068	1.56001
406	1.55695	1.56649	440	1.55337	1.56278	473	1.55061	1.55993
407	1.55684	1.56637	441	1.55327	1.56269	474	1.55054	1.55986
408	1.55672	1.56625	442	1.55318	1.56259	475	1.55046	1.55978
409	1.55660	1.56613	443	1.55309	1.56250	476	1.55039	1.55970
410	1.55648	1.56601	444	1.55300	1.56240	477	1.55031	1.55963
411	1.55637	1.56589	445	1.55291	1.56231	478	1.55024	1.55955
412	1.55625	1.56577	446	1.55282	1.56222	479	1.55017	1.55948
413	1.55614	1.56565	447	1.55273	1.56213	480	1.55010	1.55940
414	1.55603	1.56554	448	1.55264	1.56203	481	1.55003	1.55933
415	1.55592	1.56542	449	1.55255	1.56194	482	1.54995	1.55926
416	1.55580	1.56531	450	1.55247	1.56185	483	1.54988	1.55918
417	1.55569	1.56519	451	1.55238	1.56176	484	1.54981	1.55911
418	1.55558	1.56508	452	1.55229	1.56167	485	1.54974	1.55904
419	1.55548	1.56497	453	1.55221	1.56159	486	1.54967	1.55897
420	1.55537	1.56485	454	1.55212	1.56150	487	1.54961	1.55890
421	1.55526	1.56474	455	1.55204	1.56141	488	1.54954	1.55883
422	1.55515	1.56463	456	1.55195	1.56132	489	1.54947	1.55875
423	1.55505	1.56452	457	1.55187	1.56124	490	1.54940	1.55868
424	1.55494	1.56442	458	1.55179	1.56115	491	1.54933	1.55862
425	1.55484	1.56431	459	1.55171	1.56107	492	1.54927	1.55855
426	1.55474	1.56420	460	1.55162	1.56098	493	1.54920	1.55848
427	1.55463	1.56410	461	1.55154	1.56090	494	1.54913	1.55841
428	1.55453	1.56399	462	1.55146	1.56082	495	1.54907	1.55834
429	1.55443	1.56389	463	1.55138	1.56073	496	1.54900	1.55827
430	1.55433	1.56378	464	1.55130	1.56065	497	1.54894	1.55821
431	1.55423	1.56368	465	1.55122	1.56057	498	1.54887	1.55814
432	1.55413	1.56358	466	1.55115	1.56049	499	1.54881	1.55807
433	1.55403	1.56348						

λ/nm	n_o	n_e	λ/nm	n_o	n_e	λ/nm	n_o	n_e
500	1.54875	1.55801	534	1.54678	1.55597	567	1.54518	1.55432
501	1.54868	1.55794	535	1.54673	1.55592	568	1.54514	1.55427
502	1.54862	1.55788	536	1.54667	1.55587	569	1.54509	1.55423
503	1.54856	1.55781	537	1.54662	1.55581	570	1.54505	1.55418
504	1.54850	1.55775	538	1.54657	1.55576	571	1.54500	1.55414
505	1.54843	1.55768	539	1.54652	1.55570	572	1.54496	1.55409
506	1.54837	1.55762	540	1.54647	1.55565	573	1.54492	1.55405
507	1.54831	1.55756	541	1.54642	1.55560	574	1.54487	1.55400
508	1.54825	1.55749	542	1.54637	1.55555	575	1.54483	1.55396
509	1.54819	1.55743	543	1.54632	1.55549	576	1.54479	1.55391
510	1.54813	1.55737	544	1.54627	1.55544	577	1.54474	1.55387
511	1.54807	1.55731	545	1.54622	1.55539	578	1.54470	1.55383
512	1.54801	1.55725	546	1.54617	1.55534	579	1.54466	1.55378
513	1.54795	1.55718	547	1.54612	1.55529	580	1.54462	1.55374
514	1.54789	1.55712	548	1.54607	1.55524	581	1.54458	1.55370
515	1.54783	1.55706	549	1.54602	1.55519	582	1.54453	1.55365
516	1.54777	1.55700	550	1.54597	1.55514	583	1.54449	1.55361
517	1.54772	1.55694	551	1.54592	1.55509	584	1.54445	1.55357
518	1.54766	1.55688	552	1.54587	1.55504	585	1.54441	1.55352
519	1.54760	1.55682	553	1.54583	1.55499	586	1.54437	1.55348
520	1.54754	1.55676	554	1.54578	1.55494	587	1.54433	1.55344
521	1.54749	1.55671	555	1.54573	1.55489	588	1.54429	1.55340
522	1.54743	1.55665	556	1.54568	1.55484	589	1.54425	1.55336
523	1.54738	1.55659	557	1.54564	1.55479	590	1.54421	1.55331
524	1.54732	1.55653	558	1.54559	1.55474	591	1.54417	1.55327
525	1.54726	1.55648	559	1.54554	1.55470	592	1.54413	1.55323
526	1.54721	1.55642	560	1.54550	1.55465	593	1.54409	1.55319
527	1.54715	1.55636	561	1.54545	1.55460	594	1.54405	1.55315
528	1.54710	1.55631	562	1.54541	1.55455	595	1.54401	1.55311
529	1.54705	1.55625	563	1.54536	1.55451	596	1.54397	1.55307
530	1.54699	1.55619	564	1.54531	1.55446	597	1.54393	1.55303
531	1.54694	1.55614	565	1.54527	1.55441	598	1.54389	1.55299
532	1.54688	1.55608	566	1.54522	1.55437	599	1.54385	1.55295
533	1.54683	1.55603						

λ/nm	n_o	n_e	λ/nm	n_o	n_e	λ/nm	n_o	n_e
600	1.54382	1.55291	634	1.54260	1.55165	667	1.54157	1.55059
601	1.54378	1.55287	635	1.54257	1.55162	668	1.54154	1.55056
602	1.54374	1.55283	636	1.54254	1.55159	669	1.54151	1.55053
603	1.54370	1.55279	637	1.54250	1.55155	670	1.54148	1.55050
604	1.54366	1.55275	638	1.54247	1.55152	671	1.54145	1.55047
605	1.54363	1.55271	639	1.54244	1.55148	672	1.54143	1.55044
606	1.54359	1.55267	640	1.54241	1.55145	673	1.54140	1.55041
607	1.54355	1.55264	641	1.54237	1.55142	674	1.54137	1.55038
608	1.54351	1.55260	642	1.54234	1.55138	675	1.54134	1.55035
609	1.54348	1.55256	643	1.54231	1.55135	676	1.54131	1.55032
610	1.54344	1.55252	644	1.54228	1.55132	677	1.54128	1.55029
611	1.54340	1.55248	645	1.54224	1.55128	678	1.54125	1.55026
612	1.54337	1.55245	646	1.54221	1.55125	679	1.54123	1.55023
613	1.54333	1.55241	647	1.54218	1.55122	680	1.54120	1.55020
614	1.54330	1.55237	648	1.54215	1.55119	681	1.54117	1.55017
615	1.54326	1.55233	649	1.54212	1.55115	682	1.54114	1.55014
616	1.54322	1.55230	650	1.54209	1.55112	683	1.54111	1.55011
617	1.54319	1.55226	651	1.54206	1.55109	684	1.54109	1.55009
618	1.54315	1.55222	652	1.54202	1.55106	685	1.54106	1.55006
619	1.54312	1.55219	653	1.54199	1.55102	686	1.54103	1.55003
620	1.54308	1.55215	654	1.54196	1.55099	687	1.54100	1.55000
621	1.54305	1.55211	655	1.54193	1.55096	688	1.54098	1.54997
622	1.54301	1.55208	656	1.54190	1.55093	689	1.54095	1.54994
623	1.54298	1.55204	657	1.54187	1.55090	690	1.54092	1.54992
624	1.54294	1.55201	658	1.54184	1.55087	691	1.54090	1.54989
625	1.54291	1.55197	659	1.54181	1.55083	692	1.54087	1.54986
626	1.54287	1.55193	660	1.54178	1.55080	693	1.54084	1.54983
627	1.54284	1.55190	661	1.54175	1.55077	694	1.54081	1.54980
628	1.54280	1.55186	662	1.54172	1.55074	695	1.54079	1.54978
629	1.54277	1.55183	663	1.54169	1.55071	696	1.54076	1.54975
630	1.54274	1.55179	664	1.54166	1.55068	697	1.54073	1.54972
631	1.54270	1.55176	665	1.54163	1.55065	698	1.54071	1.54969
632	1.54267	1.55172	666	1.54160	1.55062	699	1.54068	1.54967
633	1.54264	1.55169				700	1.54066	1.54964