Optical Measurements

In this experiment, we will measure the optical properties of samples at the highest achievable accuracy using the available equipment.

Note: under your table you can find 2 big water bottles that you need to use for the second experiment - **do not drink them**.

In part A, we will use two different methods to measure the refractive index of a transparent disk. The first method is traditional, while the second one is original and enables a higher level of accuracy.

In part B, we will measure the ratio between the laser wavelength λ and the lattice constant of a diffraction grating d, aiming at achieving the highest possible level of accuracy.

In part C, we will measure the refractive index of a triangular prism, again – attempting to reach the highest level of accuracy.

For the experiments, the exam hall will be darkened for 100 minutes, starting 20 minutes after the exam begins (you may use a table lamp when needed). It is more convenient to perform the measurements of part A in the dark, but it is still possible to perform most of them in light as well.

You may use the walls of your cubicle as a screen, and apply adhesive tapes to the cubicle walls.

In these experiments, you are using a diode laser as a light source.

Laser safety instructions:

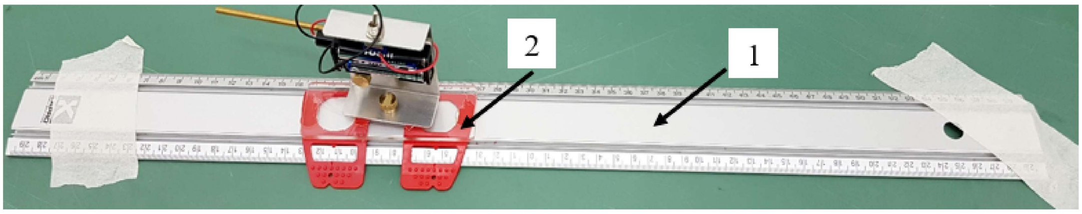
- **NEVER look directly into the laser beam!**
- In all experiments, the laser beam is horizontal. When measuring the position of the laser beam on a surface **make sure your head is ALWAYS above the level of the beam.**
- Do not aim the laser beam toward the opening of the experimental cubicle.
- Turn the laser off using the designated switch whenever you are not performing measurements.

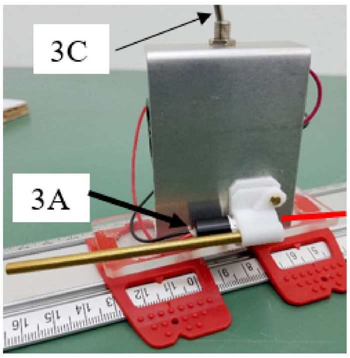
Equipment list

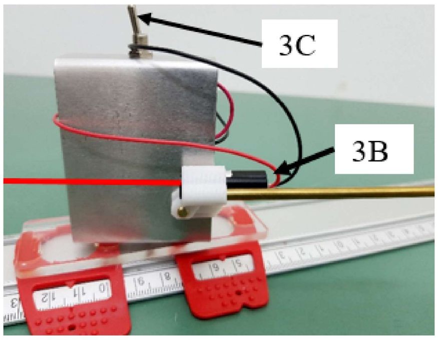
Pieces of equipment 1-9 are used in all parts of the question, and pieces of equipment 10-12 are used in separate parts of the questions. Note that you are given multiple pieces of optical equipment - make sure not to touch their vertical sides directly, to avoid fouling their surfaces.

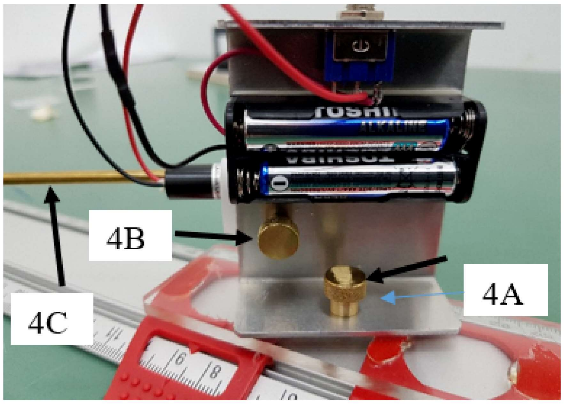
1. Ruler, 60 cm long
2. Slider that can move along the ruler
3. Laser source, mounted on the slider. The laser can be set at two heights, or levels: low level 3A for part A and high level 3B for parts B and C. The on/off switch of the laser is shown in the figure as 3C
4. The tension of screws 4A and 4B controls the resistance to rotation and therefore the stability of the apparatus. Use the small metal bar 4C to change the direction of the laser. Rotate 4C by 180 degrees to change the height of the laser. Do not rotate the laser around the beam axis, since the beam polarisation is pre-tuned.
5. Screen: you may use the cubicle walls; you may assume the walls are perpendicular to each other
6. A roll of adhesive tape which may be used to affix equipment to the table
7. Flexible measuring tape
8. A variety of rulers
9. Table lamp

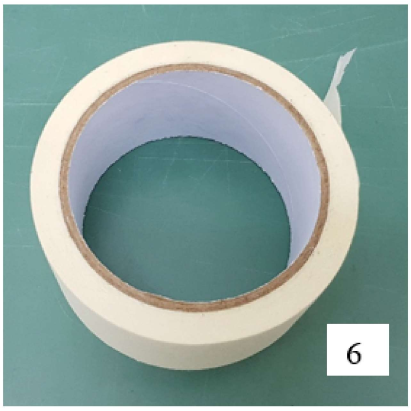
10. Round transparent disk with a diameter of 20.00 cm, fixed on a protractor, which is glued to a wooden base (for part A). You should remove the 4 small wooden cubes attached to the wooden base
11. Parchment paper that can be used as a transparent screen to be temporarily attached (by hand) to the side of the disk, allowing measuring the exit point without fouling the polished disk surface (for part A). It is more accurate to measure the beam exit point if you draw a line on the paper as shown in the figure.
12. A wooden piece (12A) and a cylinder holder (12B) capable of rotating along its vertical axis, for mounting the diffraction grating (12C) or the triangular prism (12D)



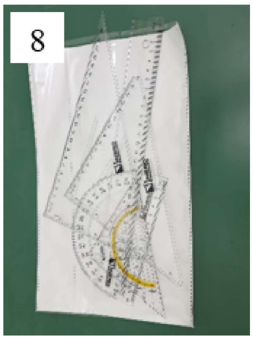





4B
4A
4C


6


7
Easy fit
3m
13mm
POWER TAPE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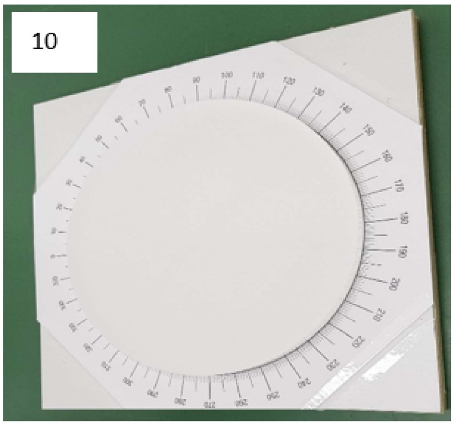
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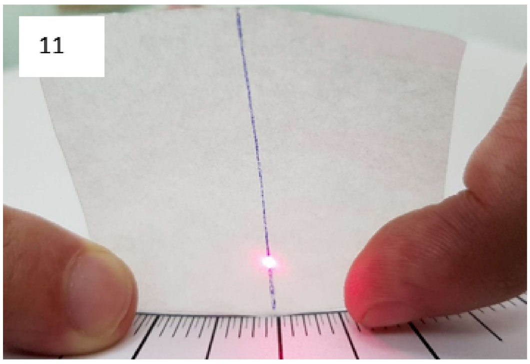
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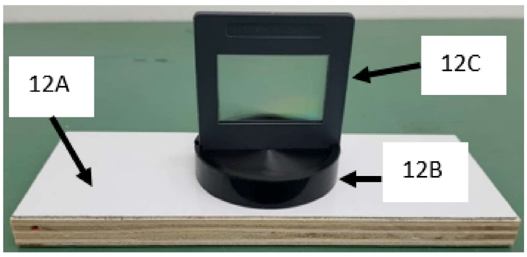
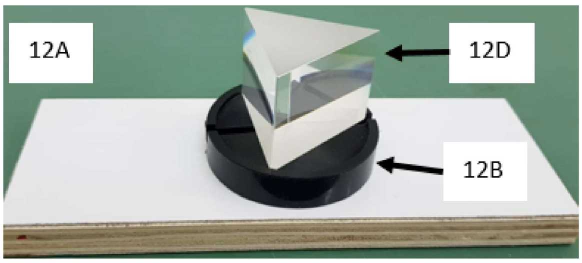
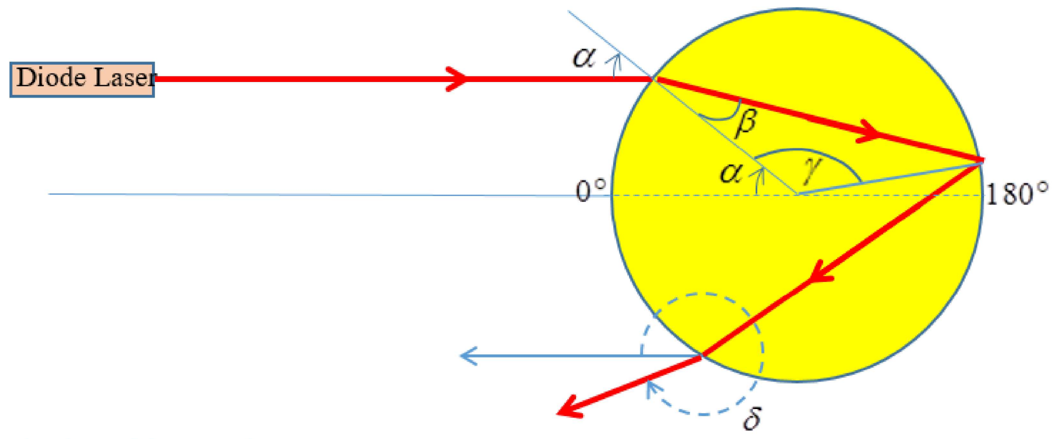

12A
12C
12B


12A
12D
12B

Part A: The refractive index of a disk (5.5 points)

In this part, we will measure the refractive index of a transparent disk by observing the path taken by a beam of light as it is refracted and reflected inside the disk.



A schematic view of the experiment

Definitions and symbols:

α	The incident angle between the disk and the incoming beam
$2\Delta\alpha$	The angular spread of the incident angle, i.e. the size of the range of values of the incident angle α
β	The refraction angle inside the disk
γ	$= 180^\circ - 2\beta$
n	The refractive index of the disk material
N	The number of times the beam hits the disk's boundary before emerging from the disk into the air (in the sketch, $N = 3$)
δ	The angle between the direction opposite to that of the incoming beam and the direction of the outgoing beam, measured clockwise (the sketch shows the angle δ for $N = 3$)
$2\Delta\delta$	The angular spread of δ

It is possible to show that the angles α, β and δ are related by:

$$\delta = 2\alpha + (N-1)(180^\circ - 2\beta) \ . \tag{1}$$

You may use this equation without deriving it.

Affix the ruler to the table in order to control the incident angle of the laser beam, using the adhesive tape, and adjust the laser in a way that will allow you to easily measure the incident angle. Then, affix the disk to the table by attaching the adhesive tape to the corners of the exposed wooden base and the table. Adjust inclination with the metal bar 4C. The laser may be set to two different heights: the low level for part A and the high level for parts B and C. The laser was tuned in advance so that the incident beam will be in the S polarization (a polarization in which the reflection is higher). **Do not change the polarization** of the incident beam (do not rotate the laser around the beam axis)!

A.1	Draw a sketch of the system showing the ruler with the slider, the disk, and the path taken by the laser beam. Indicate the incident angle α. Perform a series of measurements between the angles $15^\circ \leq \alpha \leq 75^\circ$, and write down $\alpha, \Delta\alpha, \delta, \Delta\delta$ in Table 1. Note: for measuring δ, it is more convenient to measure $\delta/2$ directly on the disk.	1.0pt

A.2	Using the measurement from the previous step, draw the appropriate graph, from which you can extract the refractive index n and the error Δn. Should you need to calculate additional quantities, fill the calculated numbers in the empty columns of Table 1. Find n and Δn.	1.0pt

A.3	For the measurements performed in A1, draw a graph of δ as a function of α. Mark on each measured point the values of $\Delta\delta$ and $\Delta\alpha$ using bars. Make an additional measurement to accurately find the minimal δ, and the corresponding α. Denote them by $\delta_{\min}$ and $\alpha_{\min}$. In order to identify the minimum point most accurately, you may use the walls of your cubicle as a screen for the outgoing beam.	0.5pt

A second method to measure the refractive index

In this section, you will develop an alternative method that will allow you to get very accurate results. Even though you are required to measure at the highest achievable accuracy, **you are not required to perform error calculations**. Nevertheless, you are required to detail the equations you use to obtain your results. Write them in the answer sheets.

A.4	Based on the behavior of the graph you got in A3, make a choice of the optimal angle to perform the measurements to find the refractive index. Write down the equation which one could use to obtain the refractive index using the developed method.	0.7pt

A.5	For $N = 3$, perform the necessary measurements to calculate the refractive index at high accuracy, using the method developed in A4 • Draw a diagram of the disk and the beam path, and indicate on the diagram the quantities you have measured. • Document the measurements you have performed. • Perform an analysis of the measurements and calculate the refractive index n of the disk, at the highest achievable accuracy. You may use the additional supplied graph pages if needed.	0.8pt

A.6 Repeat the process you performed in the previous task, for $N = 4$ and $N = 5$ (no need to draw the system and beam path). **1.5pt**

- Document the measurements you have performed at $N = 4$.
- Perform an analysis of the measurements at $N = 4$, and calculate the refractive index n using these measurements at the highest achievable accuracy.
- Document the measurements you have performed at $N = 5$.
- Perform an analysis of the measurements at $N = 5$, and calculate the refractive index n using these measurements at the highest achievable accuracy.
- From the results you obtained of the value of the refractive index using the measurements at $N = 3$, $N = 4$ and $N = 5$, calculate the average value $\langle n \rangle$ of the refractive index.

Part B: The parameters of a diffraction grating (2.5 points)

In this part you are not required to perform error calculations.

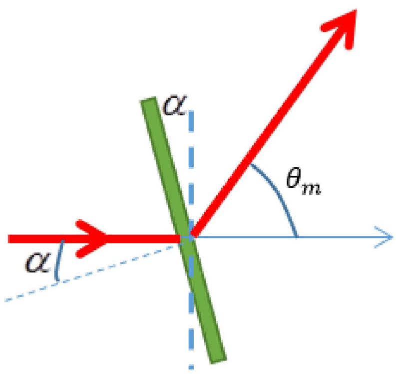
In this part we will find the ratio λ/d, where λ is the wavelength of the laser and d is the grating constant (distance between adjacent slits).

When a laser beam passes through a diffraction grating, the angle θ_m between the incident direction of the beam and the direction in which a maximal intensity (of order m) is obtained, is given by:

$$d \cdot \left(\sin\ \alpha + \sin(\theta_m - \alpha)\right) = m\lambda \quad (2)$$

where

m	the diffraction order
α	the incident angle between the beam and the grating
θ_m	the angle between the original direction of the beam and the direction in which a maximum of order m is obtained
d	the grating constant – the distance between the centers of adjacent slits in the grating



High diffraction orders allow a better separation between wavelengths. Therefore, an accurate measurement using a high diffraction order reduces the relative error in the value of λ/d.

Release the screw 4B and change the height (vertical level) of the laser, rotating the laser by 180 degrees about the horizontal axis perpendicular to the beam direction (be careful with the wires) to the state shown in 3B. This action will enable you to perform parts B and C. Use the metal bar 4C, for fine adjustment of the laser so that it will align with the height of the apparatus for measurements using a diffraction grating. Align the laser beam so that it will be perpendicular to the screen. Place the diffraction grating in the slot in the designated holder 12B. The orientation of the diffraction grating is denoted by a sticker attached to one side of the grating. Make sure that the side of the grating with the sticker is facing the laser and that the sticker is at the top of the grating. Each grating has a unique ID, written on the sticker. **Write the ID of your grating in the corresponding box in the answer sheets.**

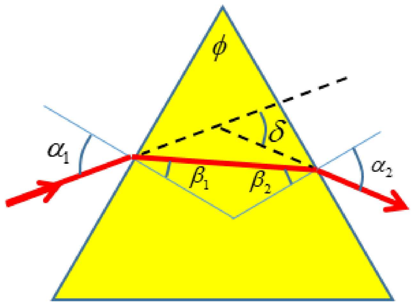
Throughout this part, you may find it useful to use an idea similar to the one you have used in the second section of part A.

B.1	• Draw on the answer sheet a diagram of the setup. In the diagram, indicate the laser on the table, the diffraction grating, the trace of the laser beam, the points it hits the screen and the quantities you measured. • Perform measurements for $m = 1$. Write down the values you measured. Extract the ratio λ/d. • Perform measurements for $m = 2$. Write down the values you measured. Extract the ratio λ/d.	0.7pt

B.2	Obtain the ratio λ/d using higher diffraction orders ($m > 2$). • Draw on the answer sheet two diagrams of the setup, for $m = 3$ and for $m = 4$. Indicate in the diagram the laser, the diffraction grating, the trace of the laser beam, the points it hits the screen and the quantities you measured. • Perform measurements for each of the orders $m = 3, 4$. Write down the values you measured. For each m, extract the measured ratio λ/d.	1.8pt

Part C: The refractive index of a triangular prism (2.0 points)

You are given an approximately equilateral, triangular prism. The three faces of the prism are planar and highly polished. The angles of the prism might deviate from 60° but by no more than 0.7° . You don't need to measure the angles of the prism. The goal of this section is to measure the refractive index of the material the prism is made of. To reduce the error in the refractive index, it is possible, by using the small angle approximation ($\sin x \approx x$, $\cos x \approx 1$ when x is measured in radians) to correct for the small deviations in the prism's angles. In this section **you are required to make error calculations**. The figures shows an example of a ray entering the prism through one face and exiting through the next one.



Place the slide ruler at a suitable place on the table so that the laser will allow you to achieve the highest accuracy in the measurements.

Place the prism in the designated holder 12B.

C.1	In the symmetric case, $\alpha_1 = \alpha_2$, the following relation holds for an equilateral prism: $n = 2\sin(\delta_{\text{sym}}/2 + 30^\circ)$. • Develop a method that would allow you to find the prism's refractive index with the highest accuracy. • Detail in the answer sheets the formulas you are using to find the refractive index.	0.4pt

C.2	• Record in the answer sheets the quantities you have measured and their values (including errors). • Calculate the refractive index of the prism for the wavelength of the laser, and the error in that value.	1.6pt