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Solutions to Experimental Problem 1

Paper transistor

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v1.4

Sketch of the solutions:

Part A. Circuit dimensioning (2.4 points)

A.1

Using Ohm's law, the current through the voltage divisor is $I = V_{\text{in}}/(R_x + R_y)$, and $V_{\text{out}} = R_y I$. Thus

A.1 0.2pt

$$V_{\text{out}} = V_{\text{in}} \frac{R_y}{R_x + R_y}$$

A.2

A.2 Uncertainty in each measurement: ±0.01 Ω 0.5pt

#	R_{T1}	R_{T2}	R_{T3}
1	122.3	125.3	125.3
2	122.3	125.4	125.4
3	122.3	125.3	125.4
4	122.2	125.2	125.5
5	122.3	125.4	125.4
6	122.3	125.4	125.3
7	122.2	125.4	125.4
8	122.2	125.3	125.4
9	122.2	125.4	125.4
10	122.2	125.4	125.5
$\overline{R}$	122.25	125.35	125.40
σ_R	0.05	0.07	0.07

A.3

A.3	For a parallelepiped conductor of length l, width w and thickness t, the resistance is given by $$R = \rho \frac{l}{w\,t}$$ For a thin film of square shape, $l = w$, thus $$R = \rho \frac{\cancel{l}}{t\cancel{w}} = \frac{\rho}{t} = R_{\square}.$$	0.3pt

A.4

The weighted average value (weighed by $1/\sigma^2$) of the sheet resistance is $\overline{R} = 123.94 \pm 0.04\,\Omega$ and $\rho = R_{\square} t$.

A.4	$\overline{R} = 123.94 \pm 0.04\,\Omega$ $\rho = 2.5 \pm 0.1 \times 10^{-3}\,\Omega$ m.	0.4pt

A.5

A.5	For a rectangular thin film $R = R_{\square}\frac{l}{w}$, thus $$R_1 = R_2 = R_{\square}\,(1 + 1/0.9 + 1/0.8 + 1/0.7 + 1/0.6 + 1/0.5 + 1/0.4 + 1/0.3) = 14.2897 R_{\square}$$ Measured values: $$\begin{aligned} R_1 &= 1776 \pm 1\Omega \quad k_1 = 14.33 \\ R_2 &= 1787 \pm 1\Omega \quad k_2 = 14.42 \\ & \overline{\kappa} = 14.3 \pm 0.1 \end{aligned}$$ Comparison with the theoretical value: the average value is compatible, within the assigned error bar, with the theoretical value.	0.5pt

A.6

A.6 Uncertainty in resistance measurements: ±1 Ω. 0.3pt

Resistor R_1:

Points	R_x/Ω	R_y/Ω
Z	1776	0
A	1708	165
B	1578	296
C	1421	452
D	1239	607
E	1033	829
F	768	1072
G	439	1394
V	0	1782

Resistor R_2:

Points	R_x/Ω	R_y/Ω
Z	1791	0
H	1428	411
I	1120	737
J	882	996
K	670	1200
L	498	1396
M	341	1555
N	188	1719
W	0	1793

A.7

A.7 0.3pt

Points	V_{out}/V	Points	V_{out}/V
Z	0	–	—
A	-0.208	H	0.664
B	-0.435	I	1.171
C	-0.699	J	1.593
D	-1.003	K	1.939
E	-1.337	L	2.24
F	-1.756	M	2.51
G	-2.29	N	2.77
V	-2.99	W	3.00

Part B. Characteristic Curves of the JFET transistor (4.5 points)

B.1

B.1 $I_{DS} =$ 11.84±0.01 mA 0.2pt

B.2

B.2 I_{DS} currents in mA: 0.8pt

Gate/Drain	Z	H	I	J	K	L	M	N	W
Z	0	1.58	2.18	2.82	3.60	4.75	6.45	9.43	11.87
A	0	1.52	2.13	2.67	3.47	4.53	6.04	7.82	8.78
B	0	1.45	2.00	2.63	3.29	4.21	5.15	5.77	6.09
C	0	1.28	1.79	2.23	2.59	2.85	2.99	3.08	3.16
D	0	0.65	0.76	0.81	0.85	0.89	0.92	0.94	0.96
E	0	0.03	0.04	0.05	0.05	0.05	0.05	0.06	0.07
F	0	0	0	0	0	0	0	0	0
G	0	0	0	0	0	0	0	0	0
V	0	0	0	0	0	0	0	0	0

B.3

The unloaded voltage is

$$V_{\text{out}} = V_{\text{in}} \frac{R_y}{R_x + R_y}$$

and the loaded voltage is

$$V_{\text{out}}^{\text{L}} = V_{\text{in}} \frac{R'_y}{R_x + R'_y},$$

where R'_y is the equivalent resistance of the parallel association between R_y and R_{L}:

$$R'_y = \frac{R_y R_{\text{L}}}{R_y + R_{\text{L}}}.$$

Thus,

$$f = \frac{\frac{R'_y}{R_x + R'_y}}{\frac{R_y}{R_x + R_y}} = \frac{(R_x + R_y)R'_y}{(R_x + R'_y)R_y} = \frac{(R_x + R_y)\frac{R_{\text{L}}}{R_y + R_{\text{L}}}}{R_x + R_y \frac{R_{\text{L}}}{R_y + R_{\text{L}}}}$$

Note that in terms of $\eta = 1/(1 + \frac{R_y}{R_{\text{L}}})$, the factor f can be written as

$$f = \frac{(R_x + R_y)\eta}{R_x + R_y \eta}$$

When $R_L \gg R_y$, $\eta \to 1$, and $f \to 1$; when $R_L \ll R_y$, $\eta \to 0$ and $f \to 0$.

B.3 0.2pt

$$f = \frac{(R_x + R_y)\eta}{R_x + R_y\eta}$$

B.4

B.4 0.7pt

Gate: A $V_{GS} = 0$ V $R_{DS} = 50.0$

Drain	V_{out}/V	V^L_{out}/V	V_{DS}/V	I_{DS}/mA	rI/V	f
Z	0,000	0,000	0,000	0,00	0,000	1,000
H	0,664	0,105	0,089	1,58	0,016	0,158
I	1,171	0,139	0,117	2,18	0,022	0,119
J	1,593	0,181	0,153	2,82	0,028	0,114
K	1,939	0,237	0,201	3,60	0,036	0,122
L	2,240	0,315	0,267	4,75	0,048	0,140
M	2,510	0,443	0,379	6,45	0,065	0,177
N	2,770	0,724	0,630	9,43	0,094	0,261
W	3,000	3,000	2,881	11,87	0,119	1,000

B.4 cont.

0.7pt

Gate: B $V_{GS} = -0.208$ V $R_{DS} = 58.73$

Drain	V_{out}/V	V_{out}^L/V	V_{DS}/V	I_{DS}/mA	rI/V	f
Z	0.000	0.000	0.000	0.00	0.000	1.000
H	0.664	0.118	0.102	1.52	0.015	0.177
I	1.171	0.157	0.136	2.13	0.021	0.134
J	1.593	0.204	0.177	2.67	0.027	0.128
K	1.939	0.267	0.233	3.47	0.035	0.138
L	2.240	0.353	0.308	4.53	0.045	0.158
M	2.510	0.495	0.435	6.04	0.060	0.197
N	2.770	0.799	0.721	7.82	0.078	0.289
W	3.000	3.000	2.912	8.78	0.088	1.000

Gate: C $V_{GS} = -0.435$ V $R_{DS} = 72.54$

Drain	V_{out}/V	V_{out}^L/V	V_{DS}/V	I_{DS}/mA	rI/V	f
Z	0.000	0.000	0.000	0.00	0.000	1.000
H	0.664	0.136	0.122	1.45	0.015	0.205
I	1.171	0.183	0.163	2.00	0.020	0.157
J	1.593	0.239	0.213	2.63	0.026	0.150
K	1.939	0.312	0.279	3.29	0.033	0.161
L	2.240	0.411	0.369	4.21	0.042	0.184
M	2.510	0.572	0.520	5.15	0.052	0.228
N	2.770	0.907	0.850	5.77	0.058	0.328
W	3.000	3.000	2.939	6.09	0.061	1.000

B.4 cont. 0.7pt

Gate: D $V_{GS} = -0.699$ V $R_{DS} = 99.86$

Drain	V_{out}/V	V_{out}^{L}/V	V_{DS}/V	I_{DS}/mA	rI/V	f
Z	0.000	0.000	0.000	0.00	0.000	1.000
H	0.664	0.170	0.157	1.28	0.013	0.256
I	1.171	0.232	0.214	1.79	0.018	0.198
J	1.593	0.303	0.281	2.23	0.022	0.190
K	1.939	0.395	0.369	2.59	0.026	0.204
L	2.240	0.516	0.487	2.85	0.029	0.230
M	2.510	0.708	0.678	2.99	0.030	0.282
N	2.770	1.089	1.059	3.08	0.031	0.393
W	3.000	3.000	2.968	3.16	0.032	1.000

Gate: E $V_{GS} = -1.003$ V $R_{DS} = 176.3$

Drain	V_{out}/V	V_{out}^{L}/V	V_{DS}/V	I_{DS}/mA	rI/V	f
Z	0.000	0.000	0.000	0.00	0.000	1.000
H	0.664	0.245	0.238	0.65	0.007	0.369
I	1.171	0.346	0.338	0.76	0.008	0.295
J	1.593	0.454	0.446	0.81	0.008	0.285
K	1.939	0.586	0.578	0.85	0.009	0.302
L	2.240	0.754	0.745	0.89	0.009	0.337
M	2.510	1.004	0.994	0.92	0.009	0.400
N	2.770	1.451	1.441	0.94	0.009	0.524
W	3.000	3.000	2.990	0.96	0.010	1.000

B.4 cont. 1.2pt

Gate: F $V_{GS} = -1.337$ V $R_{DS} = 1111$

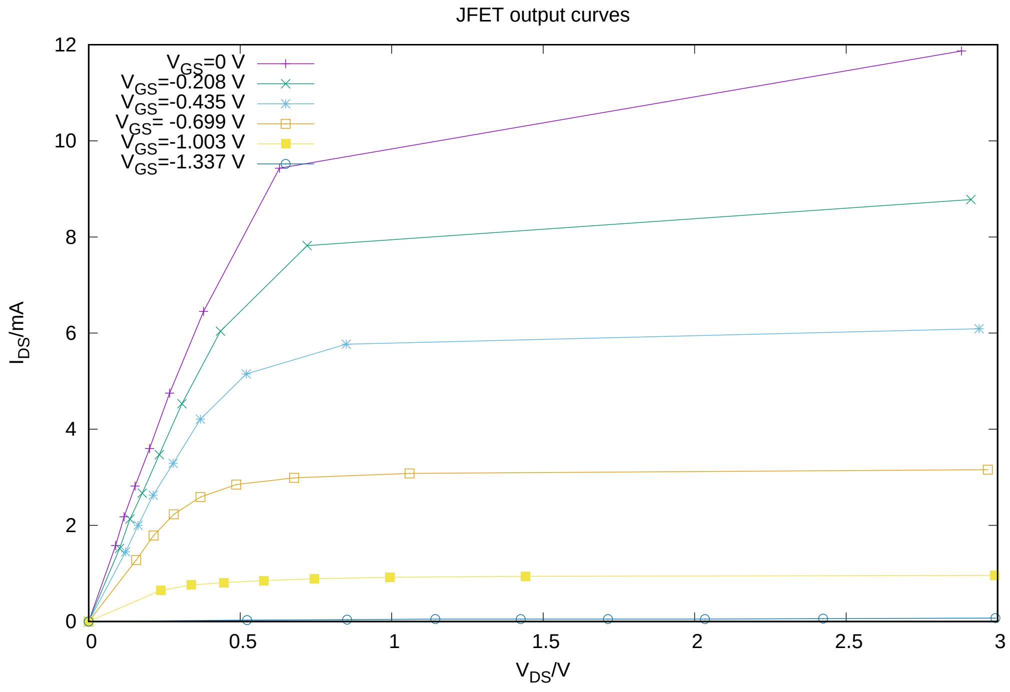
Drain	V_{out}/V	V^L_{out}/V	V_{DS}/V	I_{DS}/mA	rI/V	f
Z	0.000	0.000	0.000	0.00	0.000	1.000
H	0.664	0.526	0.523	0.03	0.003	0.791
I	1.171	0.857	0.853	0.04	0.004	0.732
J	1.593	1.149	1.144	0.05	0.005	0.721
K	1.939	1.431	1.426	0.05	0.005	0.738
L	2.240	1.719	1.714	0.05	0.005	0.767
M	2.510	2.039	2.034	0.05	0.005	0.812
N	2.770	2.430	2.424	0.06	0.006	0.877
W	3.000	3.000	2.993	0.07	0.007	1.000

Gate: G $V_{GS} = -1.756$ V $R_{DS} = \infty$

Drain	V_{out}/V	V^L_{out}/V	V_{DS}/V	I_{DS}/mA	rI/V	f
Z	0.000	0.000	0.000	0.00	0.000	1.000
H	0.664	-0.288	-0.288	0.00	0.000	-0.434
I	1.171	-0.325	-0.325	0.00	0.000	-0.278
J	1.593	-0.415	-0.415	0.00	0.000	-0.260
K	1.939	-0.562	-0.562	0.00	0.000	-0.290
L	2.240	-0.800	-0.800	0.00	0.000	-0.357
M	2.510	-1.325	-1.325	0.00	0.000	-0.528
N	2.770	-3.675	-3.675	0.00	0.000	-1.327
W	3.000	3.000	3.000	0.00	0.000	1.000

B.5

B.5 Output curves: 0.5pt



B.6

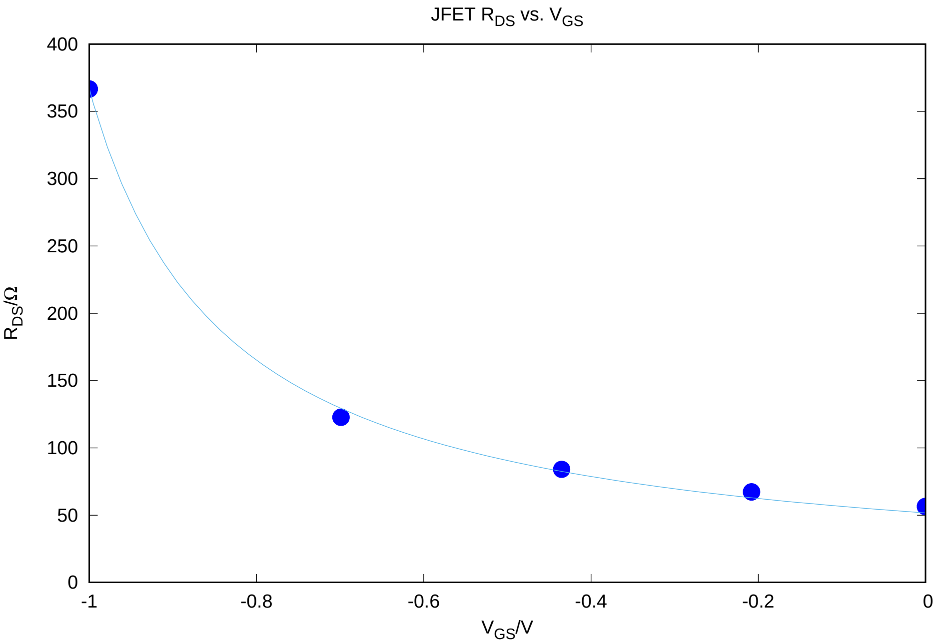
The R_{DS} values are obtained from the slopes of the linear region of the output curves (small V_{DS} voltages).

The last point in the plot $R_{\mathrm{DS}}(V_{\mathrm{GS}})$ has a large error bars as we are missing points in the linear regime, and will be ignored.

The solid line in the plot is the result of a fit to $R_{\mathrm{DS}} = R^0_{\mathrm{DS}}\,(1 - V_{\mathrm{GS}}/V_{\mathrm{P}})$, that gave $R^0_{\mathrm{DS}} = 52(2)\ \Omega$, $V_{\mathrm{P}} = -1.18(1)$ V.

B.6 0.5pt

V_{GS}/V	R_{DS}/Ω
0	56.5 ±2
-0.208	67.4 ±2
-0.435	84.1 ±4
-0.699	122.84 ±4
-1.003	366.6 ±4
-1.337	1111 ±100



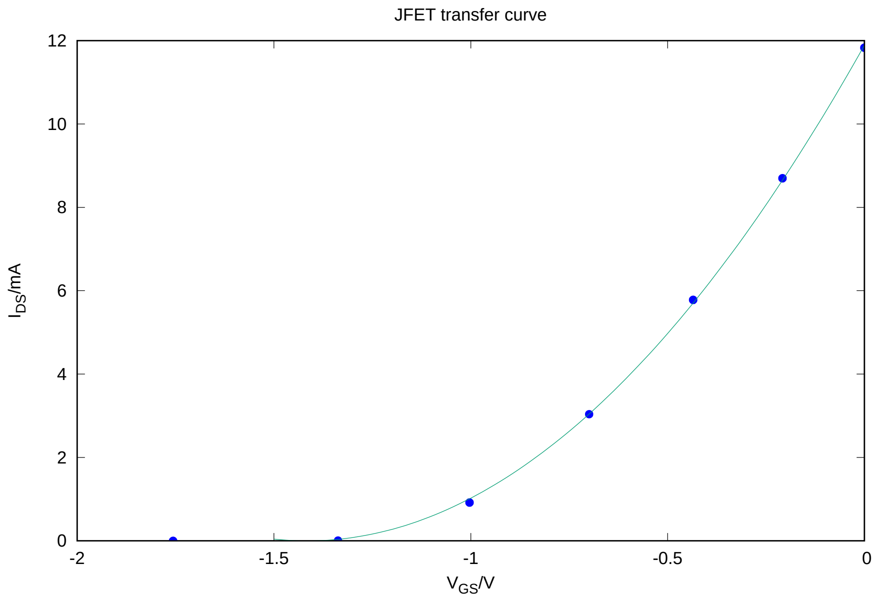
B.7

The data was obtained with $V_{DS} = +3$ V. The solid line is the result of the fit to the data of the function

$$I_{DS} = I_{DSS} \left(1 - V_{GS}/V_P\right)^2 .$$

The fitted parameters are $I_{DSS} = 11.89 \pm 0.06$ mA and $V_P = -1.42 \pm 0.02$ V.

B.7 0.3pt

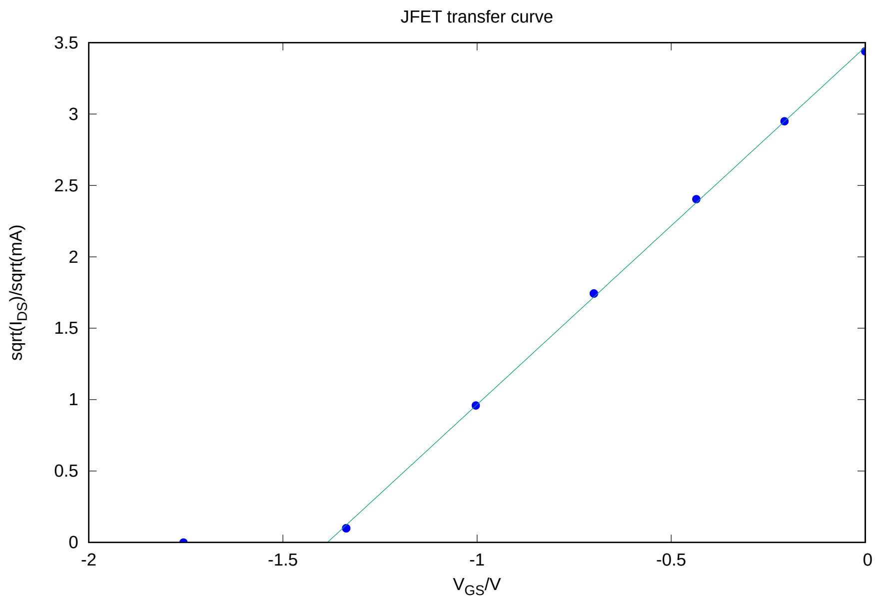


B.8

From

$$I_{DS} = I_{DSS} \left(1 - V_{GS}/V_P\right)^2$$

a plot of $\sqrt{I_{DS}}$ as function of V_{GS} should yield a straight line with slope $a = -\sqrt{I_{DS}}/V_P$ that intercepts the $x-$axis at V_P.

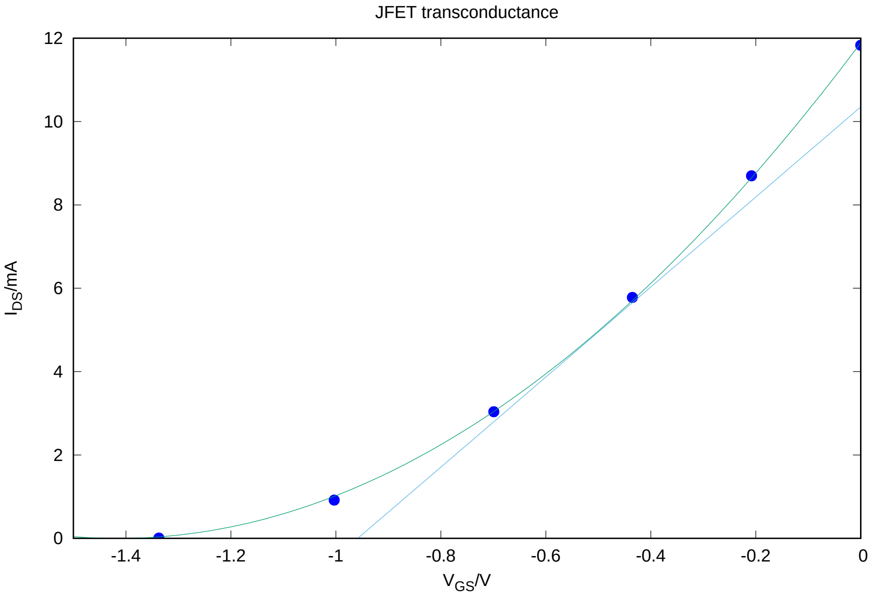


A linear fit to $f x) = a\,x + b$ gave $a = 2.50(2)$ and $b = 3.47(2)$. Thus, $V_\text{P} = -b/a = -1.39(2)$ V and $I_\text{DSS} = 4.23^2 = 12.0(2)$ mA.

B.8	$V_\text{P} = -b/a = -1.39(2)$ V $I_\text{DSS} = 4.23^2 = 12.0(2)$ mA.	0.4pt

B.9

The transcondutance is the slope of the transfer curve at a given point. From the transfer plot, we draw the tangent at the point with abscissa −0.50 V and read the slope from the graph, obtaining $g = 10.8(1)$ m $^{-1}$.



From

$$I_D = I_{DSS}\left(1 - V_{GS}/V_P\right)^2,$$

$$g = \frac{\partial I_{DS}}{\partial V_{GS}} = 2I_{DSS}\left(1 - V_{GS}/V_P\right)\left(-\frac{1}{V_P}\right) = \frac{2I_{DSS}}{V_P}\left(V_{GS}/V_P - 1\right).$$

Substituting values,

$$g = 10.8\ \text{m}^{-1}$$

a value that agrees with that obtained using the graphical method.

B.9	$g_{\text{measured}} = 10.8(1)\ \text{m}^{-1}$ $g_{\text{model}} = 10.8\ \text{m}^{-1}$	0.4pt

Part C: The Paper Thin Film Transistor (2.0 points)

C.1

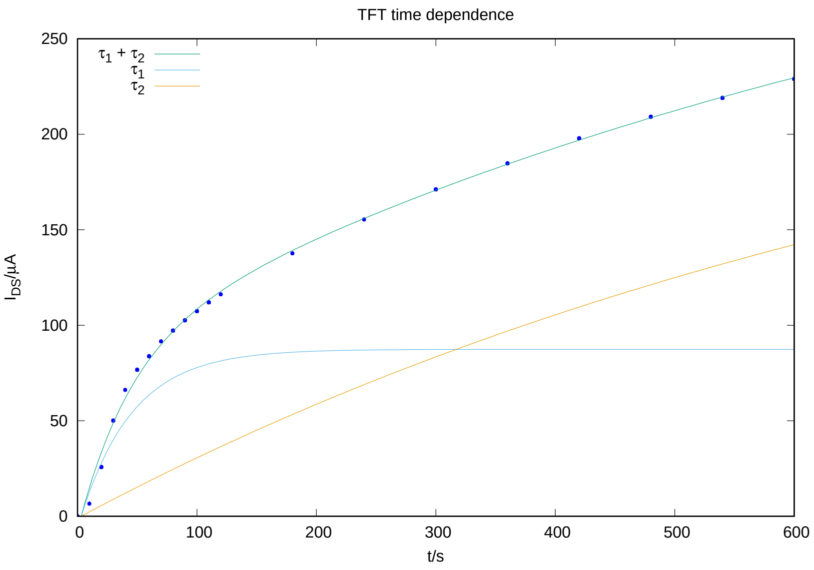
C.1 0.8pt

t/s	I_{DS}/μA	t/s	I_{DS}/μA
0	0	110	112,0
10	6.6	120	116.2
20	25.8	180	137.7
30	50.1	240	155.4
40	66.2	300	171.2
50	76.7	360	184.4
60	83.8	420	197.9
70	91.6	480	209.2
80	97.2	540	219.1
90	102.6	600	220.0
100	107.4	-	-

C.2

The data is similar to that of the charge of a capacitor, superimposed with an almost linear component that corresponds to the charge of the second capacitor with a larger time constant.

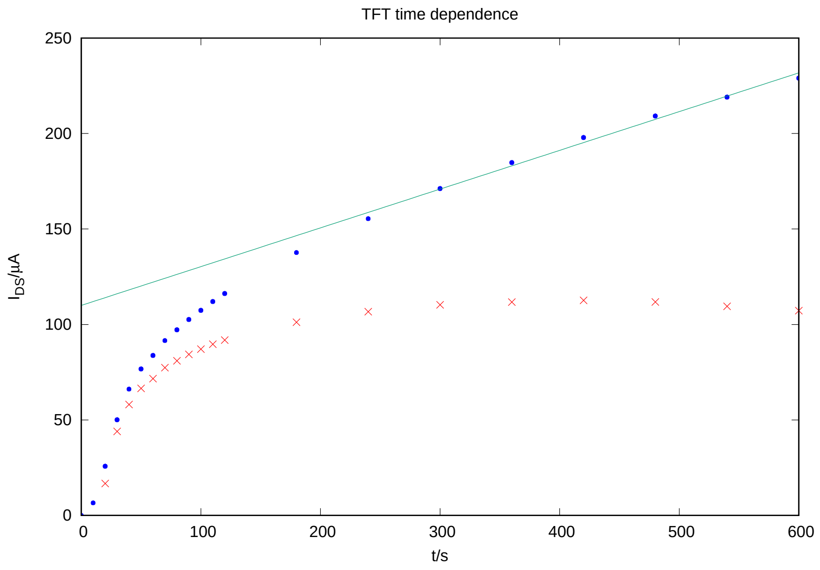
A least squares fit to a $A(1 - \exp(-t/\tau_1)) + B(1 - \exp(-t/\tau_2))$ is also depicted, showing that the data can be well fitted by this model. The shorter time constant is $\tau_1 = 43(8)$ s, the longer time constant, τ_2 is roughly 20 times larger.

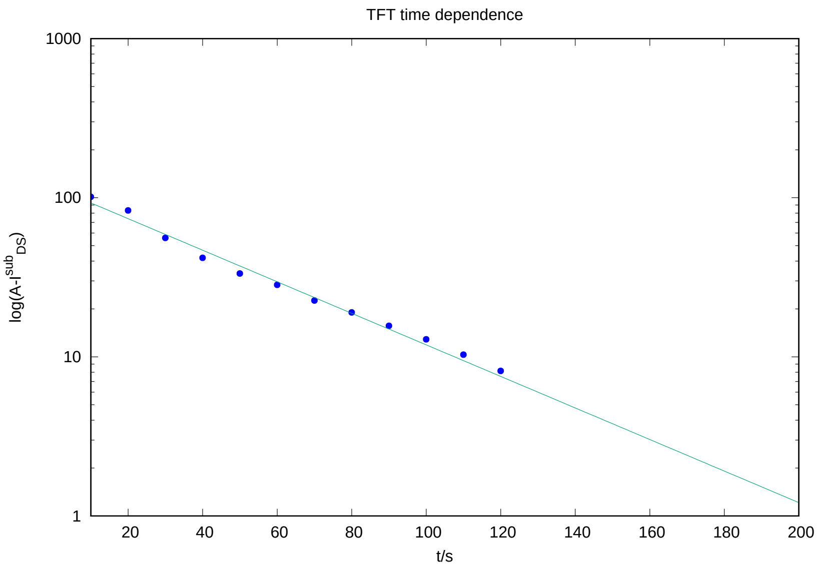


Let $I_{DS}^{sub} = A(1-\exp(-t/\tau_1))$ be the data subtracted from the long time constant component. A logarithmic plot of $\log(A - I_{DS}^{sub})$ should be a straight line of slope $-1/\tau_1$. The constant A, the saturation current of the short τ_1 component, can be easily estimated from the above plot.

The slope of the line is $m = -0.023(1)$, from which we get $\tau_1 = 44(3)$ s. The error bar is underestimated, as it does not take into account the error in the subtraction of the τ_2 component.

C.2 1.2pt





$\tau_1 = 44(3)$ s.

Part D. Inverter Circuit (1.0 points)

D.1

D.1 $R_L = 198$ kΩ 0.5pt

t	V_{in}/V	V_{out}/V
	-2.983	2.456
	-2.760	2.470
	-2.567	2.461
	-2.340	2.461
	-2.058	2.460
	-1.719	2.252
	-1.330	0.889
	-0.775	0.039

D.2

D.2 0.5pt