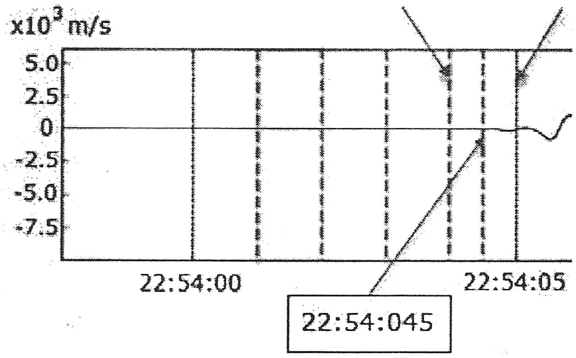
Earthquake, Volcano and Tsunami

A. Merapi Volcano Eruption

Question	Answer	Marks
A.1	Using Black's Principle the equilibrium temperature can be obtained $$m_w c_{vw}(T_e - T_w) + m_m c_{vm}(T_e - T_m) = 0$$ Thus, $$T_e = \frac{m_w c_{vw} T_w + m_m c_{vm} T_m}{m_w c_{vw} + m_m c_{vm}}$$	0.5 pts
A.2	For ideal gas, $p_e v_e = RT_e$, thus $$p_e = \frac{R}{v_e}\frac{m_w c_{vw} T_w + m_m c_{vm} T_m}{m_w c_{vw} + m_m c_{vm}}$$	0.3 pts
A.3	The relative velocity u_{rel} can be expressed as $$u_{rel} = \kappa\, p^{\alpha} V^{\beta} m^{\gamma}$$ where κ is a dimensionless constant. Using dimensional analysis, one can obtain that $$LT^{-1} = M^{\alpha+\gamma} L^{-\alpha+3\beta} T^{-2\alpha}$$ $$\alpha + \gamma = 0$$ $$-\alpha + 3\beta = 1$$ $$-2\alpha = -1$$ Therefore $$u_{rel} = \kappa\, p^{1/2} V^{1/2} m^{-1/2}$$	0.5 pts
	Total score	1.3 pts

B. The Yogyakarta Earthquake

Question	Answer	Marks	
B.1	From the given seismogram, fig. 2  One can see that the P-wave arrived at 22:54:045 or (4.5 – 5.5) seconds after the earthquake occurred at the hypocenter.	0.3 pts	0.5 pts
	Since the horizontal distance from the epicenter to the seismic station in Gamping is 22.5 km, and the depth of the hypocenter is 15 km, the distance from the hypocenter to the station is $\sqrt{22.5^2 + 15^2}$ km = 27.04 km	0.1 pts	
	Therefore, the P-wave velocity is $v_P = \frac{27.04 \text{ Km}}{4.7 \text{ s}} = 5.75 \text{ Km/s}$	0.1 pts	

<table>
<tr><th>Question</th><th>Answer</th><th colspan="2">Marks</th></tr>
<tr><td rowspan="2">B.2</td><td>Direct wave:

$$t_{\text{direct}} = \frac{SR}{v_1} = \frac{\sqrt{500^2 + 15^2}}{v_1} = \frac{502.021}{5.753}\ \text{s} = 86.9\ \text{s}$$</td><td>0.2 pts</td><td rowspan="2">0.6 pts</td></tr>
<tr><td>As in the case of an optical wave, the Snell's law is also applicable to the seismic wave.

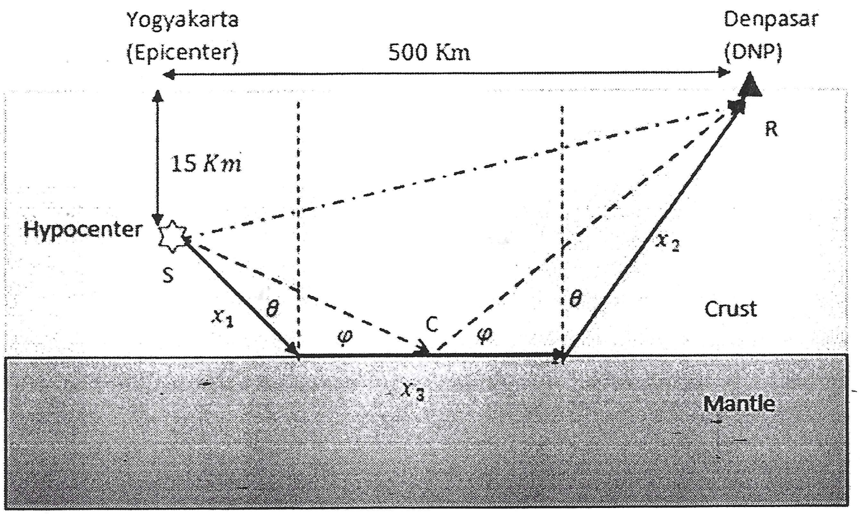


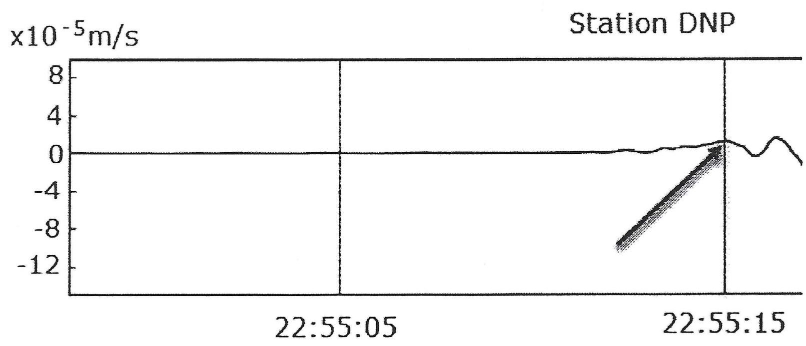
Illustration for the traveling seismic Wave

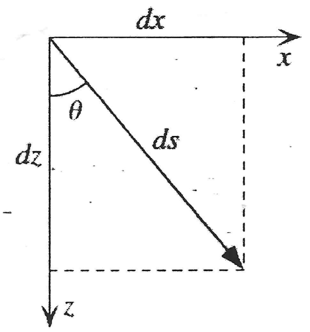
Reflected wave:

$$t_{\text{reflected}} = \frac{SC}{v_1} + \frac{CR}{v_1}$$

$$SC\cos\varphi + CR\cos\varphi = 500 \Rightarrow \cot\varphi = \frac{500}{45}$$

$$t_{\text{reflected}} = \frac{45}{v_1 \sin\varphi} = 87.3\ \text{s}$$</td><td>0.4 pts</td></tr>
</table>

Question	Answer	Marks	
B.3	Velocity of P-wave on the mantle. The fastest wave crossing the mantle is that propagating along the upperpart of the mantle. From the figure on refracted wave, we obtain that $\frac{\sin\theta}{v_1} = \frac{1}{v_2}; \quad \sin\theta = \frac{v_1}{v_2}; \quad \cos\theta = \sqrt{1-\left(\frac{v_1}{v_2}\right)^2}$ $\cos\theta = \frac{15}{x_1}; \quad x_1 = \frac{15}{\cos\theta}\text{ km}; \quad x_2 = \frac{30}{\cos\theta}\text{ km}$ $x_3 = 500-(x_1+x_2)\sin\theta = 500-45\tan\theta$	0.4 pts	1.2 pts
	The total travel time: $t = \frac{x_1+x_2}{v_1}+\frac{x_3}{v_2} = \frac{45}{v_1\cos\theta}+\frac{500}{v_2}-\frac{45\tan\theta}{v_2}$ $t\cos\theta = 45u_1 + 500u_2\cos\theta - 45u_2\sin\theta$ where $u_1 = 1/v_1$ and $u_2 = 1/v_2$. Arranging the equation, we get $(500^2+45^2)u_2^2 - 2t\ 500u_2 + t^2 - 45^2\ u_1 = 0$ whose solution is $v_2 = \frac{500tv_1^2 + 45v_1\sqrt{(45^2+500^2)-t^2v_1^2}}{t^2v_1^2-45^2}$	0.5 pts	
	 From the seismogram, we know that the fastest wave arrived at Denpasar station at 22:55:15, which is $t = 75$ s from the origin time of the earthquake in Yogyakarta. Thus $v_2 = 7.1$ km/s	0.3 pts	

Question	Answer	Marks	
B.4	By using Snell's law and defining $p=\sin\theta/v$ and $u=1/v$, we obtain $p \equiv u(0)\sin\theta_0 = u(z)\sin\theta; \qquad \sin\theta = \dfrac{p}{u(z)}$	0.2 pts	1.4 pts
	where $u(z)=1/v(z)$ and θ_0 is the initial angle of the seismic wave direction. $\dfrac{dx}{ds} = \sin\theta = \dfrac{p}{u(z)}; \qquad \dfrac{dz}{ds} = \cos\theta = \sqrt{1-\left(\dfrac{p}{u(z)}\right)^2}$ $\dfrac{dx}{dz} = \dfrac{dx}{ds}\dfrac{ds}{dz} = \dfrac{p}{u}\dfrac{u}{\left(u^2-p^2\right)^{1/2}} = p/\left(u^2-p^2\right)^{1/2}$ $x = \displaystyle\int_{z_1}^{z_2} \dfrac{p}{\left(u^2-p^2\right)^{1/2}}\, dz$	0.5 pts	
	 Illustration for the direction of wave The distance *X* is equal to twice the distance from epicenter to the turning point. The turning point is the point when $\theta = 90°$. Thus $p = u(z_t) = \dfrac{1}{v_0 + az_t}; \qquad z_t = \dfrac{1-pv_0}{ap}$ $X = 2\displaystyle\int_0^{z_t} \dfrac{p(v_0+az)}{(1-p^2(v_0+az)^2)^{1/2}} dz = \dfrac{2}{ap}\left(\sqrt{1-p^2(v_0+az)^2} - \sqrt{1-p^2v_0^2}\right)$	0.7 pts	

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<tr><td>B.5</td><td>For the travel time, $dt = \frac{ds}{v(z)}$; $\frac{dt}{ds} = u(z)$.

Thus

$$\frac{dt}{dz} = \frac{dt}{ds}\frac{ds}{dz} = \frac{u^2}{(u^2 - p^2)^{1/2}}$$
and therefore

$$T = 2\int_0^{z_t} \frac{u^2}{(u^2 - p^2)^{1/2}} dz = 2\int_0^{z_t} \frac{1}{(v_0 + az)} \frac{1}{(1 - p^2(v_0 + az)^2)^{1/2}} dz$$</td><td>1.0 pts</td><td>1.0 pts</td></tr>
<tr><td>B.6</td><td>The total travel time from the source to the Denpasar can be calculated using previous relation

$$T(p) = 2\int_0^{z_t} \frac{u^2(z)}{\left(u^2(z) - p^2\right)^{1/2}} dz$$
Which is valid for a continuous $u(z)$. For a simplified stacked of homogeneous layers (Figure F), the integral equation became a summation

$$T(p) = 2\sum_i^N \frac{u_i^2 \Delta z_i}{\left(u_i^2 - p^2\right)^{1/2}}$$</td><td>0.6 pts</td><td rowspan="2">1.0 pts</td></tr>
<tr><td></td><td>$$T(p) = 2\frac{u_1^2 \Delta z_1}{(u_1^2 - p^2)^{\frac{1}{2}}} + 2\frac{u_2^2 \Delta z_2}{(u_2^2 - p^2)^{\frac{1}{2}}} + 2\frac{u_3^2 \Delta z_3}{(u_3^2 - p^2)^{\frac{1}{2}}}$$
$$= \frac{2 \times (0.1504)^2 \times 6}{(0.1504^2 - 0.143^2)^{\frac{1}{2}}} + \frac{2 \times (0.1435)^2 \times 9}{(0.1435^2 - 0.143^2)^{\frac{1}{2}}} + \frac{2 \times (0.1431)^2 \times 15}{(0.1431^2 - 0.143^2)^{\frac{1}{2}}}$$
$$= 151.64 \text{ second}$$
Note that the actual travel time from the epicenter to Denpasar is 75 seconds. By varying the parameters of velocity and depth up to suitable value of observed travel time, physicist can know Earth structure.</td><td>0.4 pts</td></tr>
<tr><td></td><td>Total score</td><td></td><td>5.7 pts</td></tr>
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C. Java Tsunami

Question	Answer	Marks	
C.1	The center of mass of the raised ocean water with respect to the ocean surface is $h/2$. Thus $$E_P = \frac{h^2\rho\lambda Lg}{4}$$ where ρ is the ocean water density.	0.5 pts	0.5 pts
C.2	Considering a shallow ocean wave in Fig. 5, the whole water (from the surface until the ocean floor) can be considered to be moving due to the wave motion. The potential energy is equal to the kinetic energy. $$\frac{1}{4}\rho\lambda h^2 Lg = \frac{1}{4}\rho dL\lambda U^2$$ Where $x = \lambda/2$ and U is the horizontal speed of the water component. The water component that was in the upper part $hL\frac{\lambda}{2}$ should be equal to the one that moves horizontally for a half of period of time $\tau/2$, i.e. $hL\,\lambda/2 = dLU\,\tau/2$. Thus we have $$U = \frac{h\lambda}{\tau d}$$	0.7 pts	1.2 pts
	Accordingly, $\tau = \frac{\lambda}{\sqrt{gd}}$ Thus $$v = \frac{\lambda}{\tau} = \sqrt{gd}$$	0.5 pts	
C.3	Using the argument that the wave energy density is proportional to its amplitude $E = kA^2$ with A is amplitude and k is a proportional constant Because the energy flux is conserve, then $Eva = E_0 v_0 a$ for an area a where the wave flow though. Then, $kA^2\sqrt{gd} = kA_0^2\sqrt{gd_0}$ $$A = A_0\left(\frac{d_0}{d}\right)^{\frac{1}{4}}$$ (Therefore the tsunami wave will increase its amplitude and become narrower as it approaches the beach).	1.3 pts	1.3 pts
	Total score		3.0 pts

Total Score for Problem T2:

Section A :	1.3 points
Section B :	5.7 points
Section C :	3.0 points

Total : 10 points