Problem3. Simplest model of gas discharge (10 points)

An electric current flowing through a gas is called a gas discharge. There are many types of gas discharges including glow discharge in lighting lamps, arc discharge inwelding and the well known spark discharge that occurs between the clouds and the earth in the form of lightning.

Part A. Non-self-sustained gas discharge (4.8points)

In this part of the problem the so-called non-self-sustained gas discharge is studied. To maintain it a permanent operationan external ionizer is needed, which creates Z_{ext} pairs of singly ionized ions and free electrons per unit volume and per unit time uniformly in the volume.

When an external ionizer is switched on, the number of electrons and ions starts to grow. Unlimited increase in the number densities of electrons and ions in the gas is prevented by the recombination process in which a free electron recombineswith an ion to form a neutral atom. The number of recombining eventsZ_{rec} that occurs in the gas per unit volume and per unit time is given by

$$Z_{\text{rec}} = r n_e n_i,$$

where r is a constant called the recombination coefficient, and n_e,$n_i$denotethe electron and ion number densities, respectively.

Suppose that at time $t = 0$the external ionizer is switched on and the initial number densities of electrons and ions in the gas are both equal to zero. Then, the electron number density $n_e(t)$ depends on time$t$as follows:

$$n_e(t) = n_0 + a \tanh bt,$$

where n_0, a and $b$are some constants, and $\tanh x$stands for the hyperbolic tangent.

A1	Find$n_0, a, b$and express them in terms ofZ_{ext} and$r$.**(1.8points)**

Assume that there are two external ionizers available. When the first one is switched on, the electron number density in the gas reaches its equilibrium value of be $n_{e1} = 12 \cdot 10^{10}\,\text{cm}^{-3}$. When the second external ionizer is switched on, the electron number density reaches its equilibrium value of $n_{e2} = 16 \cdot 10^{10}\,\text{cm}^{-3}$.

A2	Find the electron number density$n_e$at equilibriumwhen both external ionizers are switched on simultaneously.**(0.6 points)**

Attention!*In what follows it is assumed that the external ionizer is switched on for quite long period of time such that all processes have become stationary and do not depend on time. Completely neglect the electric field due the charge carriers.*

Assume that the gas fills in the tube between the two parallel conductive plates of area S separated by the distance $L \ll \sqrt{S}$ from each other. The voltage U is applied across the plates to create an electric field between them. Assume that the number densities of both kinds of charge carriers remain almost constant along the tube.

Assumethat both the electrons (denoted by thesubscript$e$) and the ions (denoted by the subscript$i$) acquire the same ordered speed$v$due to the electric field strength$E$ found as

$$v = \beta E,$$

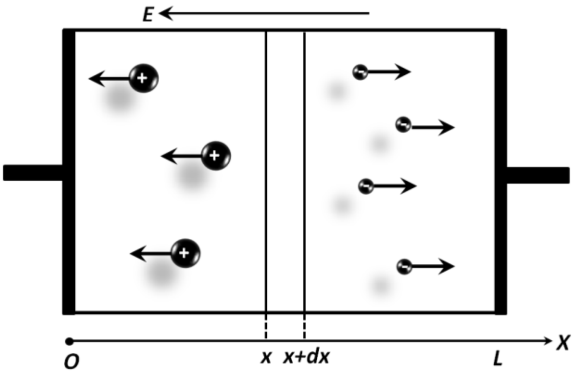
where $b$isa constant called charge mobility.

A3	Express the electric current$I$in the tube in terms of $U, \beta, L, S, Z_{\text{ext}}, r$ and $e$whichis the elementary charge.**(1.7 points)**
A4	Find the resistivityρ_{gas} of the gas at sufficiently small values of the voltage applied and express it in terms of$\beta, L, Z_{\text{ext}}, r$ and e.**(0.7 points)**

PartB. Self-sustained gas discharge (5.2 points)

In this part of the problemthe ignition of the self-sustained gas discharge is consideredto show how the electric current in the tube becomes self-maintaining.

Attention!*In the sequel assume that the external ionizer continues to operatewith the same Z_{ext} rate, neglect the electric field due to the charge carriers such that the electric field is uniform along the tube, and the recombination can be completely ignored.*



For the self-sustained gas discharge there are two important processes not considered above. The first process isasecondary electron emission, and the second one isa formation of electron avalanche. The secondary electron emission occurs when ions hit on the negative electrode, called a cathode, and the electrons are knocked out of it to move towards the positive electrode, called an anode. The ratio of the number of the knocked electrons$\dot{N}_e$per unit timeto the number of ions$\dot{N}_i$ hitting the cathode per unit time is called the coefficient of the secondary electron emission,$\gamma = \dot{N}_e/\dot{N}_i$.The formation of the electron avalanche isexplained as follows. The electric field accelerates free electrons which acquire enough kinetic energy to ionize the atoms in the gas by hitting them. As a result the number of free electrons moving towards the anodesignificantly increases. This process is described by the Townsend coefficient α, which characterizes an increase in the number of electronsdN_edue to moving N_e electrons that have passed the distancedl, i.e.

$$\frac{dN_e}{dl} = \alpha N_e.$$

The total current$I$ in any cross section of the gas tube consists of the ion $I_i(x)$and the electron$I_e(x)$ currents which, in the steady state, depend on the coordinate x, shown in the figureabove. The electron current $I_e(x)$ varies along the x-axis according to the formula

$$I_e(x) = C_1 e^{A_1 x} + A_2,$$

where A_1, A_2, C_1 are some constants.

B1	Find$A_1, A_2$and express them in terms of$Z_{\text{ext}}, \alpha, e, L, S$.**(2 points)**

The ion current $I_i(x)$ varies along the x-axis according to the formula

$$I_i(x) = C_2 + B_1 e^{B_2 x},$$

where B_1, B_2, C_2 are some constants.

B2	Find$B_1, B_2$and express them in terms of$Z_{\text{ext}}, \alpha, e, L, S, C_1$.**(0.6 points)**
B3	Write down the condition for $I_i(x)$at$x = L$.**(0.3 points)**
B4	Write down the condition for$I_i(x)$and$I_e(x)$at $x = 0$.**(0.6 points)**
B5	Find the total current$I$and express it in terms of$Z_{\text{ext}}, \alpha, \gamma, e, L, S$.Assume that it remains finite **(1.2 points)**

Let the Townsend coefficient αbe constant. When the length of the tube turns out greater than some critical value, i.e. $L > L_{\text{cr}}$, the external ionizer can be turned off and the discharge becomes self-sustained.

B6	Find$L_{cr}$and express it in terms of$Z_{\text{ext}}, \alpha, \gamma, e, L, S$.**(0.5 points)**