Problem T1. Focus on sketches *(13 points)*

Part A. Ballistics *(4.5 points)*

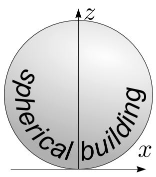
A ball, thrown with an initial speed v_0, moves in a homogeneous gravitational field in the x-z plane, where the x-axis is horizontal, and the z-axis is vertical and antiparallel to the free fall acceleration g. Neglect the effect of air drag.

i. (0.8 pts) By adjusting the launching angle for a ball thrown with a fixed initial speed v_0 from the origin, targets can be hit within the region given by

$$z \leq z_0 - kx^2.$$

You can use this fact without proving it. Find the constants z_0 and k.

ii. (1.2 pts) The launching point can now be freely selected on the ground level $z = 0$, and the launching angle can be adjusted as needed. The aim is to hit the topmost point of a spherical building of radius R (see fig.) with the minimal initial speed v_0. Bouncing off the roof prior to hitting the target is not allowed. Sketch qualitatively the shape of the optimal trajectory of the ball (use the designated box on the answer sheet). Note that the marks are given only for the sketch.



iii. (2.5 pts) What is the minimal launching speed $v_{\min}$ needed to hit the topmost point of a spherical building of radius R?

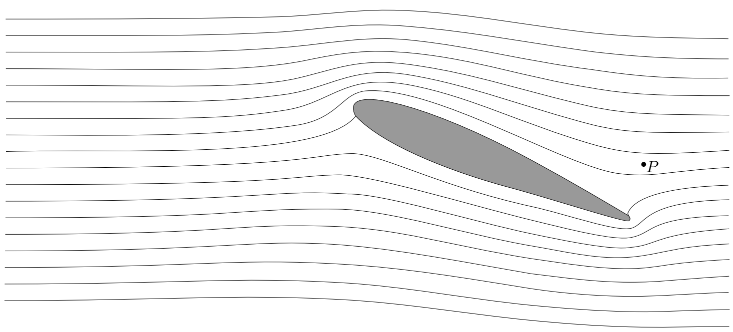
La Geode, Parc de la Villette, Paris. Photo: katchooo/flickr.com

Part B. Air flow around a wing *(4 points)*

For this part of the problem, the following information may be useful. For a flow of liquid or gas in a tube along a streamline, $p + \rho g h + \frac{1}{2}\rho v^2 = \text{const.}$, assuming that the velocity v is much less than the speed of sound. Here ρ is the density, h is the height, g is free fall acceleration and p is hydrostatic pressure. Streamlines are defined as the trajectories of fluid particles (assuming that the flow pattern is stationary). Note that the term $\frac{1}{2}\rho v^2$ is called the dynamic pressure.

In the fig. shown below, a cross-section of an aircraft wing is depicted together with streamlines of the air flow around the wing, as seen in the wing's reference frame. Assume that (a) the air flow is purely two-dimensional (i.e. that the velocity vectors of air lie in the plane of the figure); (b) the streamline pattern is independent of the aircraft speed; (c) there is no wind; (d) the dynamic pressure is much smaller than the atmospheric pressure, $p_0 = 1.0 \times 10^5\,\text{Pa}$.

You can use a ruler to take **measurements from the fig. on the answer sheet**.



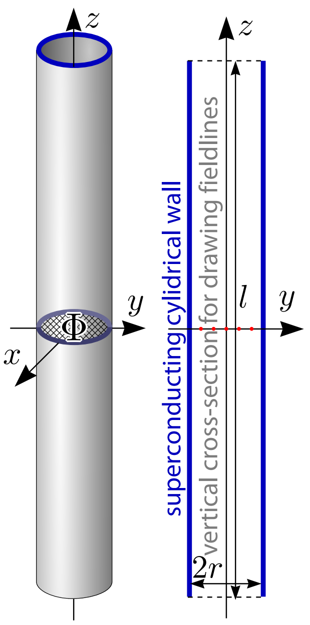
i. (0.8 pts) If the aircraft's ground speed is $v_0 = 100\,\text{m/s}$, what is the speed of the air, v_P, at the point P (marked in the fig.) with respect to the ground?

ii. (1.2 pts) In the case of high relative humidity, as the ground speed of the aircraft increases over a critical value v_{crit}, a stream of water droplets is created behind the wing. The droplets emerge at a certain point Q. Mark the point Q in the fig. on the answer sheet. Explain qualitatively (using formulae and as little text as possible) how you determined the position of Q.

iii. (2.0 pts) Estimate the critical speed v_{crit} using the following data: relative humidity of the air is $r = 90\%$, specific heat capacity of air at constant pressure $c_p = 1.00 \times 10^3\,\text{J/kg}\cdot\text{K}$, pressure of saturated water vapour: $p_{sa} = 2.31\,\text{kPa}$ at the temperature of the unperturbed air $T_a = 293\,\text{K}$ and $p_{sb} = 2.46\,\text{kPa}$ at $T_b = 294\,\text{K}$. Depending on your approximations, you may also need the specific heat capacity of air at constant volume $c_V = 0.717 \times 10^3\,\text{J/kg}\cdot\text{K}$. Note that the relative humidity is defined as the ratio of the vapour pressure to the saturated vapour pressure at the given temperature. Saturated vapour pressure is defined as the vapour pressure by which vapour is in equilibrium with the liquid.

Part C. Magnetic straws *(4.5 points)*

Consider a cylindrical tube made of a superconducting material. The length of the tube is l and the inner radius is r with $l \gg r$. The centre of the tube coincides with the origin, and its axis coincides with the z-axis.

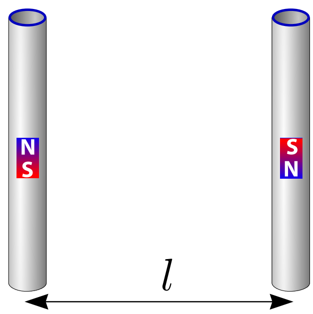


There is a magnetic flux Φ through the central cross-section of the tube, $z = 0$, $x^2 + y^2 < r^2$. A superconductor is a material which expels any magnetic field (the field is zero inside the material).

i. (0.8 pts) Sketch five such magnetic field lines, which pass through the five red dots marked on the axial cross-section of the tube, on the designated diagram on the answer sheet.

ii. (1.2 pts) Find the tension force T along the z-axis in the middle of the tube (i.e. the force by which two halves of the tube, $z > 0$ and $z < 0$, interact with each other).

iii. (2.5 pts) Consider another tube, identical and parallel to the first one.



The second tube has the same magnetic field but in the opposite direction and its centre is placed at $y = l$, $x = z = 0$ (so that the tubes form opposite sides of a square). Determine the magnetic interaction force F between the two tubes.