1. Image of a charge in a metallic object

Introduction – Method of images

A point charge q is placed in the vicinity of a grounded metallic sphere of radius R [see Fig. 1(a)], and consequently a surface charge distribution is induced on the sphere. To calculate the electric field and potential from the distribution of the surface charge is a formidable task. However, the calculation can be considerably simplified by using the so called method of images. In this method, the electric field and potential produced by the charge distributed on the sphere can be represented as an electric field and potential of a single point charge q' placed inside the sphere (you do not have to prove it). Note: **The electric field of this image charge q' reproduces the electric field and the potential only outside the sphere (including its surface)**.

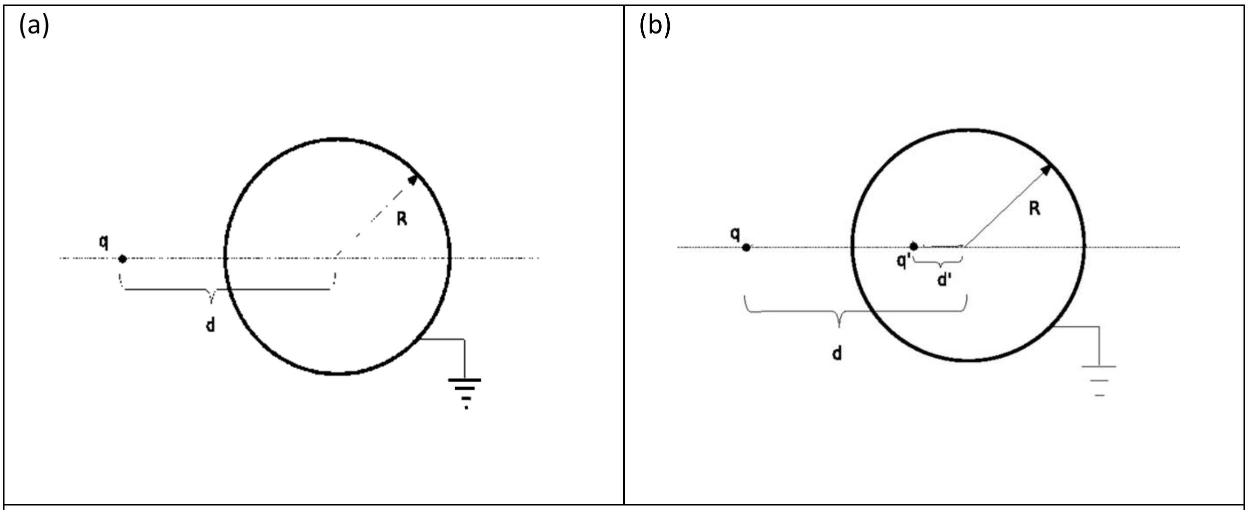

Fig 1. (a) A point charge q in the vicinity of a grounded metallic sphere. (b) The electric field of the charge induced on the sphere can be represented as electric field of an image charge q'.

Task 1 – The image charge

The symmetry of the problem dictates that the charge q' should be placed on the line connecting the point charge q and the center of the sphere [see Fig. 1(b)].

a) What is the value of the potential on the sphere? *(0.3 points)*

b) Express q' and the distance d' of the charge q' from the center of the sphere, in terms of q, d, and R. *(1.9 points)*

c) Find the magnitude of force acting on charge q. Is the force repulsive? *(0.5 points)*

Task 2 – Shielding of an electrostatic field

Consider a point charge *q* placed at a distance *d* from the center of a grounded metallic sphere of radius *R*. We are interested in how the grounded metallic sphere affects the electric field at point *A* on the opposite side of the sphere (see Fig. 2). Point *A* is on the line connecting charge *q* and the center of the sphere; its distance from the point charge *q* is *r*.

a) Find the vector of the electric field at point *A*. *(0.6 points)*

b) For a very large distance $r >> d$, find the expression for the electric field by using the approximation $(1+a)^{-2} \approx 1-2a$, where $a << 1$. *(0.6 points)*

c) In which limit of *d* does the grounded metallic sphere screen the field of the charge *q* completely, such that the electric field at point *A* is exactly zero? *(0.3 points)*

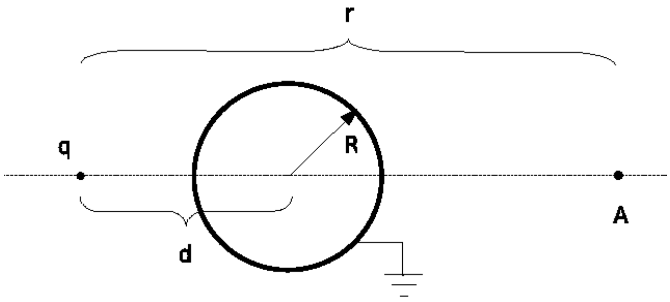

Fig 2. The electric field at point *A* is partially screened by the grounded sphere.

Task 3 – Small oscillations in the electric field of the grounded metallic sphere

A point charge *q* with mass *m* is suspended on a thread of length *L* which is attached to a wall, in the vicinity of the grounded metallic sphere. In your considerations, ignore all electrostatic effects of the wall. The point charge makes a mathematical pendulum (see Fig. 3). The point at which the thread is attached to the wall is at a distance l from the center of the sphere. Assume that the effects of gravity are negligible.

a) Find the magnitude of the electric force acting on the point charge *q* for a given angle α and indicate the direction in a clear diagram *(0.8 points)*

b) Determine the component of this force acting in the direction perpendicular to the thread in terms of l, *L, R, q and* α. *(0.8 points)*

c) Find the frequency for small oscillations of the pendulum. *(1.0 points)*

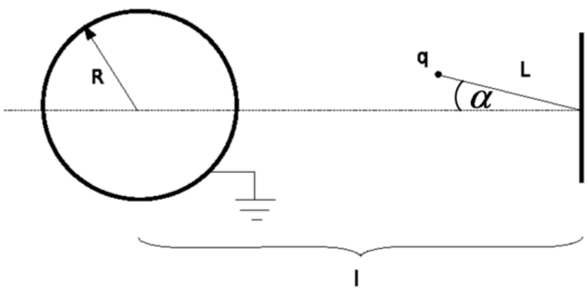

Fig 3. A point charge in the vicinity of a grounded sphere oscillates as a pendulum.

Task 4 – The electrostatic energy of the system

For a distribution of electric charges it is important to know the electrostatic energy of the system. In our problem (see Fig. 1a), there is an electrostatic interaction between the external charge *q* and the induced charges on the sphere, and there is an electrostatic interaction among the induced charges on the sphere themselves. In terms of the charge q, radius of the sphere R and the distance d determine the following electrostatic energies:

a) the electrostatic energy of the interaction between charge q and the induced charges on the sphere; *(1.0 points)*
b) the electrostatic energy of the interaction among the induced charges on the sphere; *(1.2 points)*
c) the total electrostatic energy of the interaction in the system. *(1.0 points)*

Hint: There are several ways of solving this problem:

(1) In one of them, you can use the following integral,

$$\int_d^\infty \frac{xdx}{\left(x^2-R^2\right)^2}=\frac{1}{2}\frac{1}{d^2-R^2}.$$

(2) In another one, you can use the fact that for a collection of N charges q_i located at points $\vec{r}_i, i=1,\ldots,N$, the electrostatic energy is a sum over all pairs of charges:

$$V=\frac{1}{2}\sum_{i=1}^{N}\sum_{\substack{j=1\\ i\neq j}}^{N}\frac{1}{4\pi\varepsilon_0}\frac{q_i q_j}{\left|\vec{r}_i-\vec{r}_j\right|}.$$