Theoretical Problem 1

A A bungee jumper is attached to one end of a long elastic rope. The other end of the elastic rope is fixed to a high bridge. The jumper steps off the bridge and falls, from rest, towards the river below. He does not hit the water. The mass of the jumper is m, the unstretched length of the rope is L, the rope has a force constant (force to produce 1 m extension) of k and the gravitational field strength is g.

You may assume that

the jumper can be regarded as a point mass m attached to the end of the rope,
the mass of the rope is negligible compared to m,
the rope obeys Hooke's law,
air resistance can be ignored throughout the fall of the jumper.

Obtain expressions for the following and insert on the answer sheet:

(a) the distance y dropped by the jumper before coming instantaneously to rest for the first time,

(b) the maximum speed v attained by the jumper during this drop,

(c) the time t taken during the drop before coming to rest for the first time.

B A heat engine operates between two identical bodies at different temperatures T_A and T_B ($T_A > T_B$), with each body having mass m and constant specific heat capacity s. The bodies remain at constant pressure and undergo no change of phase.

(a) Showing full working, obtain an expression for the final temperature T_0 attained by the two bodies A and B, if the heat engine extracts from the system the maximum amount of mechanical work that is theoretically possible.

Write your expression for the final temperature T_0 on the answer sheet.

(b) Hence, obtain and write on the answer sheet an expression for this maximum amount of work available.

The heat engine operates between two tanks of water each of volume 2.50 m^3. One tank is at 350 K and the other is at 300 K.

(c) Calculate the maximum amount of mechanical energy obtainable. Insert the value on the answer sheet.

Specific heat capacity of water = 4.19×10^3 J kg^{-1} K^{-1}

Density of water = 1.00×10^3 kg m^{-3}

C It is assumed that when the earth was formed the isotopes ${}^{238}U$ and ${}^{235}U$ were present but not their decay products. The decays of ${}^{238}U$ and ${}^{235}U$ are used to establish the age of the earth, *T*.

(a) The isotope ${}^{238}U$ decays with a half-life of 4.50×10^9 years. The decay products in the resulting radioactive series have half-lives short compared to this; to a first approximation their existence can be ignored. The decay series terminates in the stable lead isotope ${}^{206}Pb$.

Obtain and insert on the answer sheet an expression for the number of ${}^{206}Pb$ atoms, denoted ${}^{206}n$, produced by radioactive decay with time t, in terms of the present number of ${}^{238}U$ atoms, denoted ${}^{238}N$, and the half-life time of ${}^{238}U$. (You may find it helpful to work in units of 10^9 years.)

(b) Similarly, ${}^{235}U$ decays with a half-life of 0.710×10^9 years through a series of shorter half-life products to give the stable isotope ${}^{207}Pb$.

Write down on the answer sheet an equation relating ${}^{207}n$ to ${}^{235}N$ and the half-life of ${}^{235}U$.

(c) A uranium ore, mixed with a lead ore, is analysed with a mass spectrometer. The relative concentrations of the three lead isotopes ${}^{204}Pb$, ${}^{206}Pb$ and ${}^{207}Pb$ are measured and the number of atoms are found to be in the ratios 1.00 : 29.6 : 22.6 respectively. The isotope ${}^{204}Pb$ is used for reference as it is <u>not</u> of radioactive origin. Analysing a pure lead ore gives ratios of 1.00 : 17.9 : 15.5.

Given that the ratio ${}^{238}N$: ${}^{235}N$ is 137 : 1, derive and insert on the answer sheet an equation involving *T*.

(d) Assume that *T* is much greater than the half lives of both uranium isotopes and hence obtain an approximate value for *T*.

(e) This approximate value is clearly not significantly greater than the longer half life, but can be used to obtain a much more accurate value for *T*. Hence, or otherwise, estimate a value for the age of the earth correct to within 2%.

D Charge Q is uniformly distributed *in vacuo* throughout a spherical volume of radius R.

(a) Derive expressions for the electric field strength at distance r from the centre of the sphere for $r \leq R$ and $r > R$.

(b) Obtain an expression for the total electric energy associated with this distribution of charge.

Insert your answers to (a) and (b) on the answer sheet.

E A circular ring of thin copper wire is set rotating about a vertical diameter at a point within the Earth's magnetic field. The magnetic flux density of the Earth's magnetic field at this point is 44.5 μT directed at an angle of 64^{o} below the horizontal. Given that the density of copper is 8.90×10^{3} kg m^{-3} and its resistivity is 1.70×10^{-8} Ω m, calculate how long it will take for the angular velocity of the ring to halve. Show the steps of your working and insert the value of the time on the answer sheet. This time is much longer than the time for one revolution.

You may assume that the frictional effects of the supports and air are negligible, and for the purposes of this question you should ignore self-inductance effects, although these would not be negligible.