3 Faster than light?[7]

3.1 Problem text

In this problem we analyze and interpret measurements made in 1994 on radio wave emission from a compound source within our galaxy.

The receiver was tuned to a broad band of radio waves of wavelengths of several centimeters. Figure 3.1 shows a series of images recorded at different times. The contours indicate constant radiation strength in much the same way as altitude contours on a geographical map. In the figure the two maxima are interpreted as showing two objects moving away from a common center shown by crosses in the images. (The center, which is assumed to be fixed in space, is also a strong radiation emitter but mainly at other wavelengths). The measurements conducted on the various dates were made at the same time of day.

The scale of the figure is given by a line segment showing one arc second (as). (1 as = 1/3600 of a degree). The distance to the celestial body at the center of the figure, indicated by crosses, is estimated to be $R = 12.5$ kpc. A kiloparsec (kpc) equals $3.09 \cdot 10^{19}$ m. The speed of light is c = $3.00 \cdot 10^{8}$ m/s. Error calculations are not required in the solution.

a) (2 points) We denote the angular positions of the two ejected radio emitters, relative to the common center, by $\theta_1(t)$ and $\theta_2(t)$, where the subscripts 1 and 2 refer to the left and right hand ones, respectively, and t is the time of observation. The angular speeds, as seen from the Earth, are ω_1 and ω_2. The corresponding apparent transverse linear speeds of the two sources are denoted by $v'_{1,\perp}$ and $v'_{2,\perp}$.

Using Figure 3.1, make a graph to find the numerical values of ω_1 and ω_2 in milli-arc-seconds per day (mas/d). Also determine the numerical values of $v'_{1,\perp}$ and $v'_{2,\perp}$, and write all answers on the answer sheet. (You may be puzzled by some of the results).

b) (3 points) In order to resolve the puzzle arising in part (a), consider a light-source moving with velocity $\vec{v}$ at an angle ϕ $(0 \leq \phi \leq \pi)$ to the direction towards a distant observer O (Figure 3.2). The speed may be written as $v = \beta c$, where c is the speed of light. The distance to the source, as measured by the observer, is R. The angular speed of the source, as seen from the observer, is ω, and the apparent linear speed perpendicular to the line of sight is $v'_{\perp}$.

Find ω and $v'_{\perp}$ in terms of β, R and ϕ and write your answer on the answer sheet.

c) (1 point) We assume that the two ejected objects, described in the introduction and in part (a), are moving in opposite directions with equal speeds $v = \beta c$. Then the results of part (b) make it possible to calculate β and ϕ from the angular speeds ω_1 and ω_2 and the distance R. Here ϕ is the angle defined in part (b), for the left hand object, corresponding to subscript 1 in part (a).

Derive formulas for β and ϕ in terms of known quantities and determine their numerical values from the data in part (a). Write your answers in the designated fields on the answer sheet.

d) (2 points) In the one-body situation of part (b), find the condition for the apparent perpendicular speed $v'_{\perp}$ to be larger than the speed of light c.

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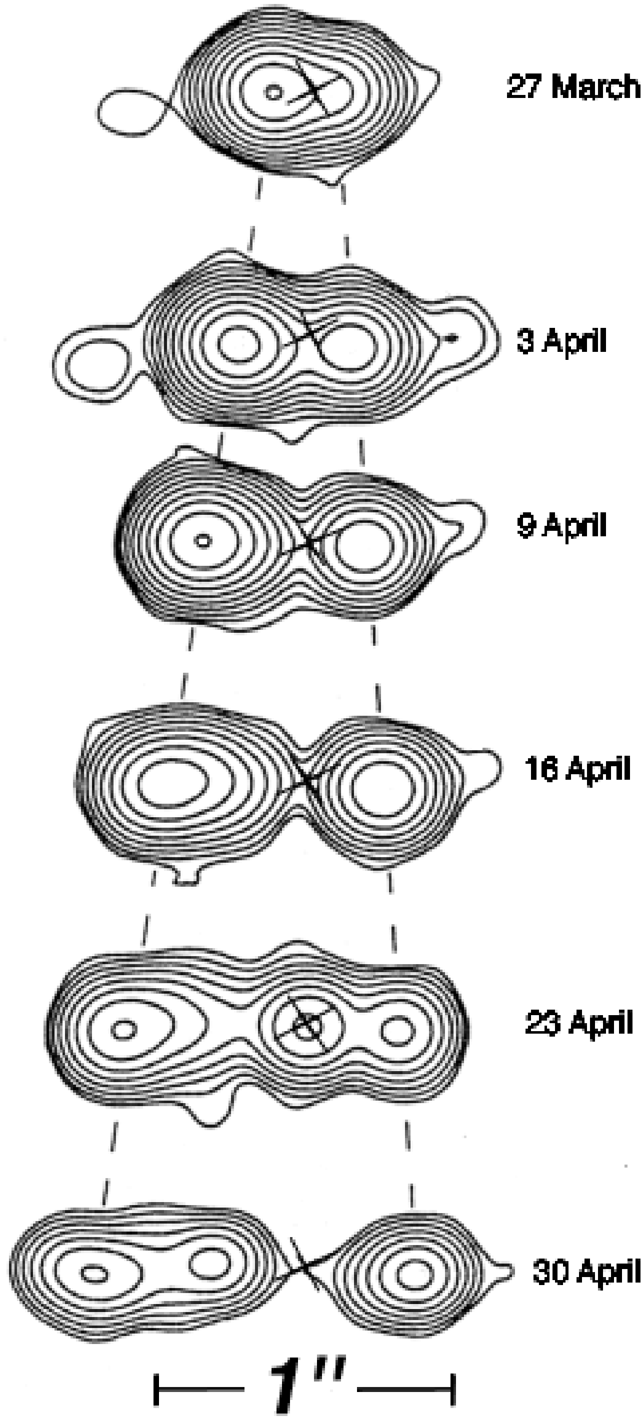

Figure 3.1: *Radio emission from a source in our galaxy.*

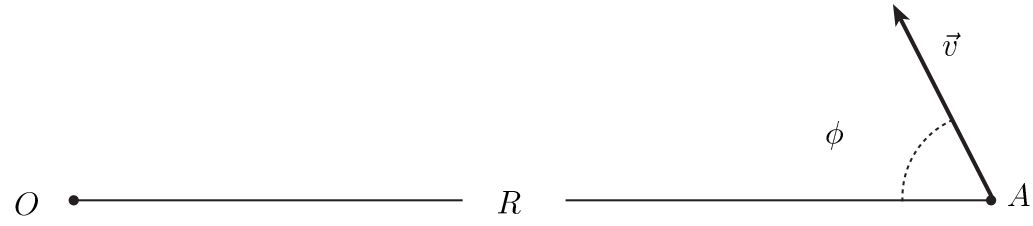

Figure 3.2: *The observer is at O and the original position of the light source is at A. The velocity vector is $\vec{v}$.*

Write the condition in the form $\beta > f(\phi)$ and provide an analytic expression for the function f on the answer sheet.

Draw on the graph answer sheet the physically relevant region of the (β, ϕ)-plane. Show by shading in which part of this region the condition $v'_\perp > c$ holds.

e) (1 point) Still in the one-body situation of part (b), find an expression for the maximum value $(v'_\perp)_{max}$ of the apparent perpendicular speed $v'_\perp$ for a given β and write it in the designated field on the answer sheet. Note that this speed increases without limit when $\beta \to 1$.

f) (1 point) The estimate for R given in the introduction is not very reliable. Scientists have therefore started speculating on a better and more direct method for determining R. One idea for this goes as follows. Assume that we can identify and measure the Doppler shifted wavelengths λ_1 and λ_2 of radiation from the two ejected objects, corresponding to the same known original wavelength λ_0 in the rest frames of the objects.

Starting from the equations for the relativistic Doppler shift, $\lambda = \lambda_0(1-\beta\cos\phi)(1-\beta^2)^{-1/2}$, and assuming, as before, that both objects have the same speed, v, show that the unknown $\beta = v/c$ can be expressed in terms of λ_0, λ_1, and λ_2 as

$$\beta = \sqrt{1 - \frac{\alpha\,\lambda_0^2}{(\lambda_1+\lambda_2)^2}}\ . \tag{3.1}$$

Write the numerical value of the coefficient α in the designated field on the answer sheet.

You may note that this means that the suggested wavelength measurements will in practice provide a new estimate of the distance.

3.2 Solution

a) On Figure 3.1 we mark the centers of the sources as neatly as we can. Let $\theta_1(t)$ be the angular distance of the left center from the cross as a function of time and $\theta_2(t)$ the angular distance of the right center. We measure these quantities on the figure at the given times by a ruler and convert to arcseconds according to the given scale. This results in the following numerical data: