1 Rolling of a hexagonal prism[1]

1.1 Problem text

Consider a long, solid, rigid, regular hexagonal prism like a common type of pencil (Figure 1.1). The mass of the prism is M and it is uniformly distributed. The length of each side of the cross-sectional hexagon is a. The moment of inertia I of the hexagonal prism about its central axis is

$$I = \frac{5}{12} M a^2 \tag{1.1}$$

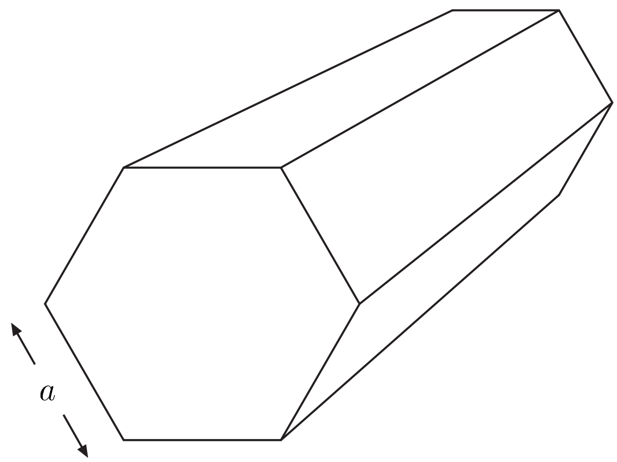

Figure 1.1: *A solid prism with the cross section of a regular hexagon.*

The moment of inertia I' about an edge of the prism is

$$I' = \frac{17}{12} M a^2 \tag{1.2}$$

a) (3.5 points) The prism is initially at rest with its axis horizontal on an inclined plane which makes a small angle θ with the horizontal (Figure 1.2). Assume that the surfaces of the prism are slightly concave so that the prism only touches the plane at its edges. The effect of this concavity on the moment of inertia can be ignored. The prism is now displaced from rest and starts an uneven rolling down the plane. Assume that friction prevents any sliding and that the prism does not lose contact with the plane. The angular velocity just before a given edge hits the plane is ω_i while ω_f is the angular velocity immediately after the impact.

Show that we may write

$$\omega_f = s\omega_i \tag{1.3}$$

and write the value of the coefficient s on the answer sheet.

[1]Authors: Leó Kristjánsson and Thorsteinn Vilhjálmsson

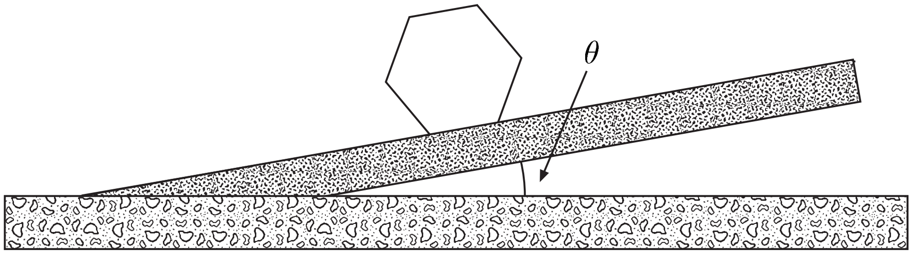

Figure 1.2: *A hexagonal prism lying on an inclined plane.*

b) (1 point) The kinetic energy of the prism just before and after impact is similarly K_i and K_f.

Show that we may write

$$K_f = rK_i \tag{1.4}$$

and write the value of the coefficient r on the answer sheet.

c) (1.5 points) For the next impact to occur K_i must exceed a minimum value $K_{i,min}$ which may be written in the form

$$K_{i,min} = \delta Mga \tag{1.5}$$

where $g = 9.81$ m/s^2 is the acceleration of gravity.

Find the coefficient δ in terms of the slope angle θ and the coefficient r. Write your answer on the answer sheet. (Use the algebraic symbol r, not its value).

d) (2 points) If the condition of part (c) is satisfied, the kinetic energy K_i will approach a fixed value $K_{i,0}$ as the prism rolls down the incline.

Given that the limit exists, show that $K_{i,0}$ may be written as:

$$K_{i,0} = \kappa Mga \tag{1.6}$$

and write the coefficient κ in terms of θ and r on the answer sheet.

e) (2 points) Calculate, to within 0.1°, the minimum slope angle θ_0, for which the uneven rolling, once started, will continue indefinitely. Write your numerical answer on the answer sheet.

1.2 Solution

a)
Solution Method 1

At the impact the prism starts rotating about a new axis, i.e. the edge which just hit the plane. The force from the plane has no torque about this axis, so that the angular momentum about the edge is conserved during the brief interval of impact. The linear