Theory Question 2: Solution:

Nuclear Masses and Stability

(a) The alpha-decay process is as follows:

$$A \rightarrow (A\text{-}4) + \alpha\,(A=4)$$

Therefore the energy criterion for decay to happen is:

$$m_A - m_{A-4} - m_4 > 0$$

The number and type of nucleons in the decay is preserved so we only have to consider the binding energies:

$$- B_A + B_{A-4} + B_4 > 0$$

If we write $B/A = a + bA$, where a and b are constants to be found from the graph, then this equation becomes:

$$-A(a+bA) + (A-4)(a+b(A-4)) + B_4 > 0$$

$$- 8bA - 4a + 16b + B_4 > 0$$

By inspecting the graph, a good linear approximation to B/A above $A = 100$ is:

$$B/A = (9.6 - 0.0080 \times A)\ \text{MeV}$$

i.e. $a = 9.6$ MeV and $b = 0.0080$ MeV, and the condition becomes:

$$-0.064A - 38.4 - 0.1 + 25.0 > 0$$

$$\underline{A > 13.5/0.064 = 211}$$

Part (b).

(i) Because A is fixed we only need to consider the penultimate two terms which depend on Z.

$$\frac{dB}{dZ} = -2Za_c A^{-1/3} - \frac{a_a}{A}(-4A + 8Z)$$

$$Z_{\max} = \frac{4a_a}{2a_c A^{-1/3} + 8a_a / A} = \frac{A}{2} \frac{1}{\left(1 + \frac{a_c A^{\frac{2}{3}}}{4a_a}\right)}$$

(ii) $Z_{\max} = 79.25$

The full expression for the differential equation in (a) is:

$$\frac{dB}{dZ} = -2Za_c A^{-1/3} - \frac{a_a}{A}(-4A + 8Z) \pm 2a_p A^{-3/4}$$

The last term is positive if a change in Z of +1 changes the nucleus from an even-even to an odd-odd, and negative if the reverse is true. Note A is positive in this case.

How do we deal with the last term?

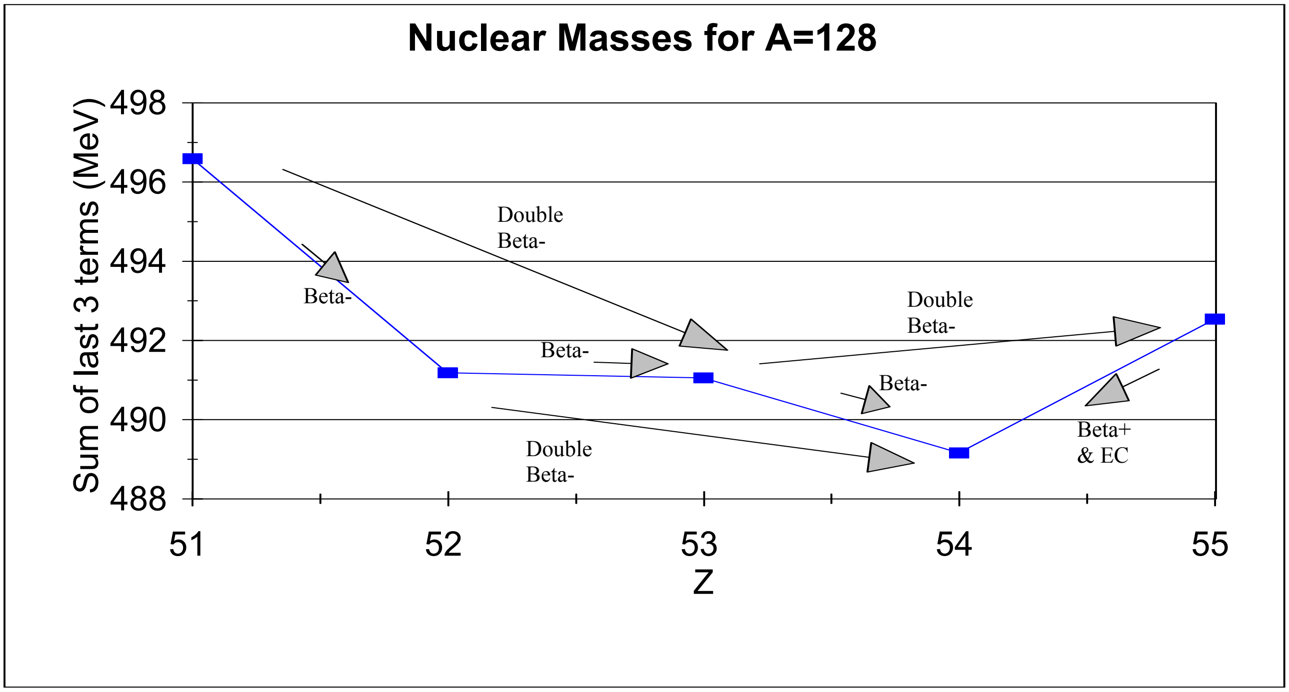
The number $Z_{\max}$ has to be an integer, and even numbers are favoured over odd so we can guess $Z_{\max}$ = 80. To check, evaluate the last three terms for various values of Z:

77	979.241
78	975.915
79	976.295
80	975.341
81	978.093
82	979.512
83	984.637

This confirms that $Z_{\max}$ = 80; this is an even-even nucleus.

(iii) Consider only the last three terms in the equation; the rest are constant if A is constant. Call the sum of these quantities X. To find out whether these nuclei are stable we need to find differences in X between neighbouring species and to compare these differences with the energy requirements for each would-be decay process.

(i) β^-- decay; $n \rightarrow p + e^-$, need ΔX > -1.30 + 0.51 = - 0.79 MeV
(ii) β^+- decay; $p \rightarrow n + e^+$, need ΔX > 1.30 + 0.51 = 1.81MeV
(iii) $\beta^-\beta^-$ - decay; $2n \rightarrow 2p + 2e^-$, need ΔX > 2(-1.30 + 0.51) = - 1.58 MeV
(iv) Electron capture; $e^- + p \rightarrow n$, need ΔX > 1.30 - 0.51 = 0.79 MeV



↑ For information: graph not expected from students

Nucleus/Process	X (MeV)
$^{128}_{51}Sb$	496.59
$^{128}_{52}Te$	491.19
$^{128}_{53}I$	491.06
$^{128}_{54}Xe$	489.16
$^{128}_{55}Cs$	492.54

↑ For information: table not expected from students

Nucleus/Process	β^- - decay	β^+ - decay	Electron-capture	$\beta^-\beta^-$ - decay
$^{128}_{53}I$	√	0	0	√
$^{128}_{54}Xe$	0	0	0	0
$^{128}_{55}Cs$	0	√	√	0

Students will fill out this table

Theory Question No.2: Mark Distribution

Smallest fractional mark allowed: 0.25

Marks allowed for errors consistently propagated only if physically reasonable.

(a)	Approach	1.5	
	Correct Answer	1.5	
			3
(b)(i)	Approach	1	
	Correct Answer	1	
			2
(b)(ii)	Approach	1	
	Correct Answer	1	
			2
(b)(iii)	0.25 for each of 12 entries	3	
			3
	Grand Total		10