

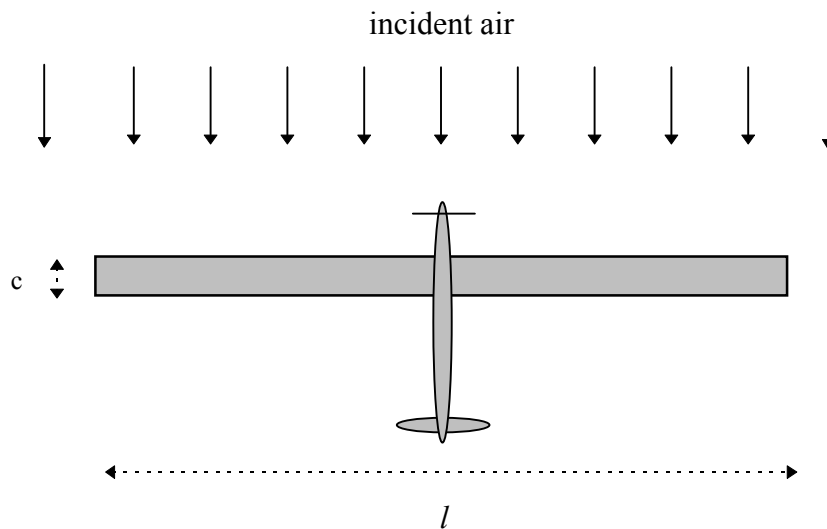
Theory Question No.3

Solar-Powered Aircraft

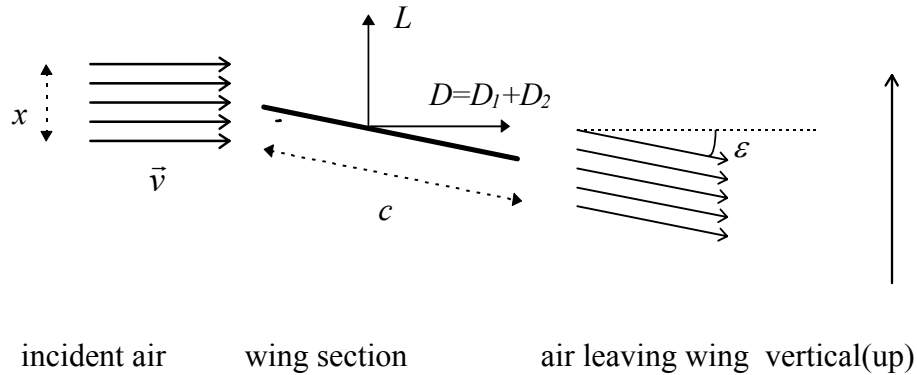
We wish to design an aircraft which will stay aloft using solar power alone. The most efficient type of layout is one with a wing whose top surface is completely covered in solar cells. The cells supply electrical power with which the motor drives the propeller.

Consider a wing of rectangular plan-form with span l , chord (width) c ; the wing area is $S = cl$, and the wing aspect ratio $A = l/c$. We can get an approximate idea of the wing's performance by considering a slice of air of height x and length l being deflected downward at a small angle ε with only a very small change in speed. Control surfaces can be used to select an optimal value of ε for flight. This simple model corresponds closely to reality if $x = \pi l/4$, and we can assume this to be the case. The total mass of the aircraft is M and it flies horizontally with velocity \bar{v} relative to the surrounding air. In the following calculations consider only the air flow around the wing.

Top view of aircraft (in its own frame of reference):



Side view of wing (in a frame of reference moving with the aircraft):



Ignore the modification of the airflow due to the propeller.

(a) Consider the change in momentum of the air moving past the wing, with *no* change in speed while it does so. Derive expressions for the vertical lift force L and the horizontal drag force D_1 on the wing in terms of wing dimensions, v , ϵ , and the air density ρ . Assume the direction of air flow is always parallel to the plane of the side-view diagram. (3 marks)

(b) There is an additional horizontal drag force D_2 caused by the friction of air flowing over the surface of the wing. The air slows slightly, with a change of speed Δv ($\ll 1\%$ of v) given by:

$$\frac{\Delta v}{v} = \frac{f}{A}$$

The value of f is independent of ϵ .

Find an expression (in terms of M, f, A, S, ρ and g - the acceleration due to gravity) for the flight speed v_0 corresponding to a minimum power being needed to maintain this aircraft in flight at constant altitude and velocity. Neglect terms of order $(\epsilon^2 f)$ or higher. (3 marks)

You may find the following small angle approximation useful:

$$1 - \cos \epsilon \approx \frac{\sin^2 \epsilon}{2}$$

(c) On the answer sheet, sketch a graph of power P versus flight speed v . Show the separate contributions to the power needed from the two sources of drag. Find an expression (in terms of M, f, A, S, ρ and g) for the minimum power, P_{min} . (2 marks)

(d) If the solar cells can supply sufficient energy so that the electric motors and propellers generate mechanical power of $I = 10$ watts per square metre of wing area, calculate the maximum wing loading Mg/S (N/m^2) for this power and flight speed v_0 (m/s). Assume $\rho = 1.25 \text{ kg/m}^3$, $f = 0.004$, $A = 10$. (2 marks)