Theory Question No.2

Nuclear Masses and Stability

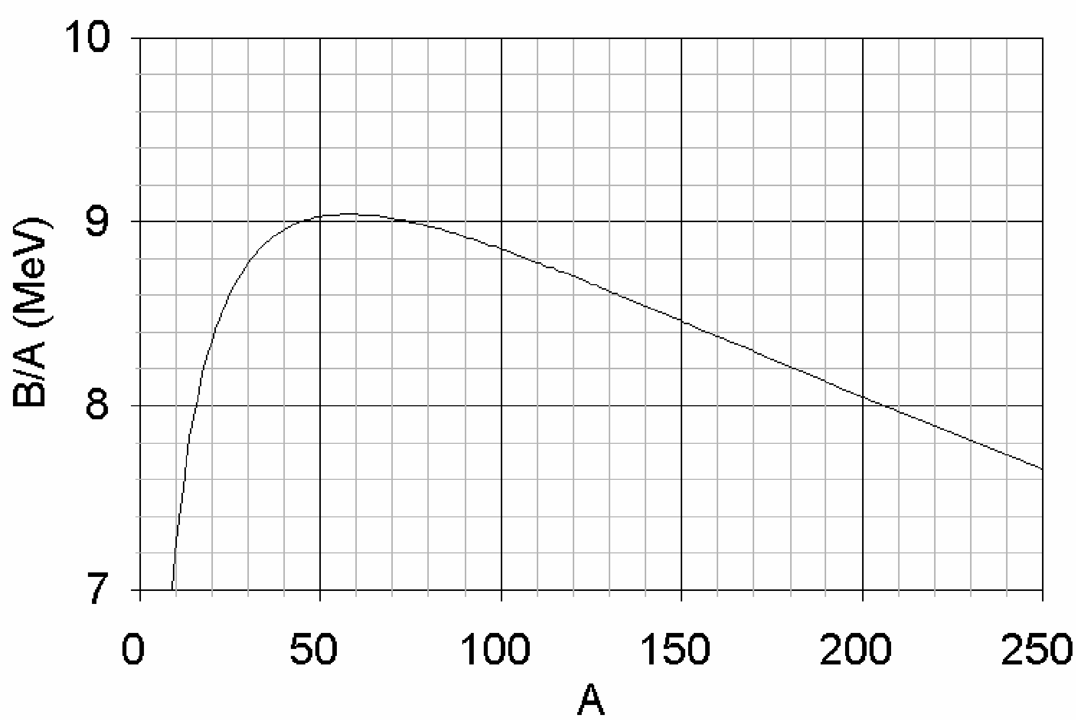
All energies in this question are expressed in MeV - millions of electron volts.
One MeV = 1.6×10^{-13} J, but it is not necessary to know this to solve the problem.

The mass M of an atomic nucleus with Z protons and N neutrons (i.e. the mass number $A = N + Z$) is the sum of masses of the free constituent nucleons (protons and neutrons) minus the binding energy B/c^2.

$$Mc^2 = Zm_pc^2 + Nm_nc^2 - B$$

The graph shown below plots the maximum value of B/A for a given value of A, vs. A. The greater the value of B/A, in general, the more stable is the nucleus.

Binding Energy per Nucleon



(a) Above a certain mass number A_α, nuclei have binding energies which are always small enough to allow the emission of alpha-particles (*A=4*). Use a linear approximation to this curve above A = 100 to estimate A_α. (3 *marks*)

For this model, assume the following:

- Both initial and final nuclei are represented on this curve.
- The total binding energy of the alpha-particle is given by B_4 = 25.0 MeV (this cannot be read off the graph!).

(b) The binding energy of an atomic nucleus with *Z* protons and *N* neutrons (*A*=*N*+*Z*) is given by a semi-empirical formula:

$$B = a_v A - a_s A^{2/3} - a_c Z^2 A^{-1/3} - a_a \frac{(N-Z)^2}{A} - \delta$$

The value of δ is given by:

$+ a_p A^{-3/4}$ for odd-N/odd-Z nuclei

0 for even-N/odd-Z or odd-N/even-Z nuclei

$- a_p A^{-3/4}$ for even-N/even-Z nuclei

The values of the coefficients are:

a_v = 15.8 MeV; a_s = 16.8 MeV; a_c = 0.72 MeV; a_a = 23.5 MeV; a_p = 33.5 MeV.

(i) Derive an expression for the proton number Z_{max} of the nucleus with the largest binding energy for a given mass number A . Ignore the δ-term for this part only. (2 *marks*)

(ii) What is the value of Z for the A = 200 nucleus with the largest B/A? Include the effect of the δ-term. (2 *marks*)

(iii) Consider the three nuclei with A = 128 listed in the table on the answer sheet. Determine which ones are energetically stable and which ones have sufficient energy to decay by the processes listed below. Determine Z_{max} as defined in part (i) and fill out the table on your answer sheet.

In filling out the table, please:

- Mark processes which are energetically allowed thus: √
- Mark processes which are NOT energetically allowed thus: 0
- Consider only transitions between these three nuclei.

Decay processes:

(1) β^-- decay; emission from the nucleus of an electron
(2) β^+- decay; emission from the nucleus of a positron
(3) $\beta^-\beta^-$ - decay; emission from the nucleus of two electrons simultaneously
(4) Electron capture; capture of an *atomic* electron by the nucleus.

The rest mass energy of an electron (and positron) is $m_e c^2 = 0.51$ MeV; that of a proton is $m_p c^2 = 938.27$ MeV; that of a neutron is $m_n c^2 = 939.57$ MeV.

(3 *marks*)