27th INTERNATIONAL PHYSICS OLYMPIAD
OSLO, NORWAY

Model Answer

for the

EXPERIMENTAL COMPETITION

JULY 4 1996

These model answers indicate what is required from the candidates to get the maximum score of 20 marks. Some times we have used slightly more text than required; paragraphs written in italic give additional comments. This practical exam will reward students with creativity, intuition and a thorough understanding of the physics involved.

Alternative solutions regarded as less elegant or more time consuming are printed in frames like this with white background.

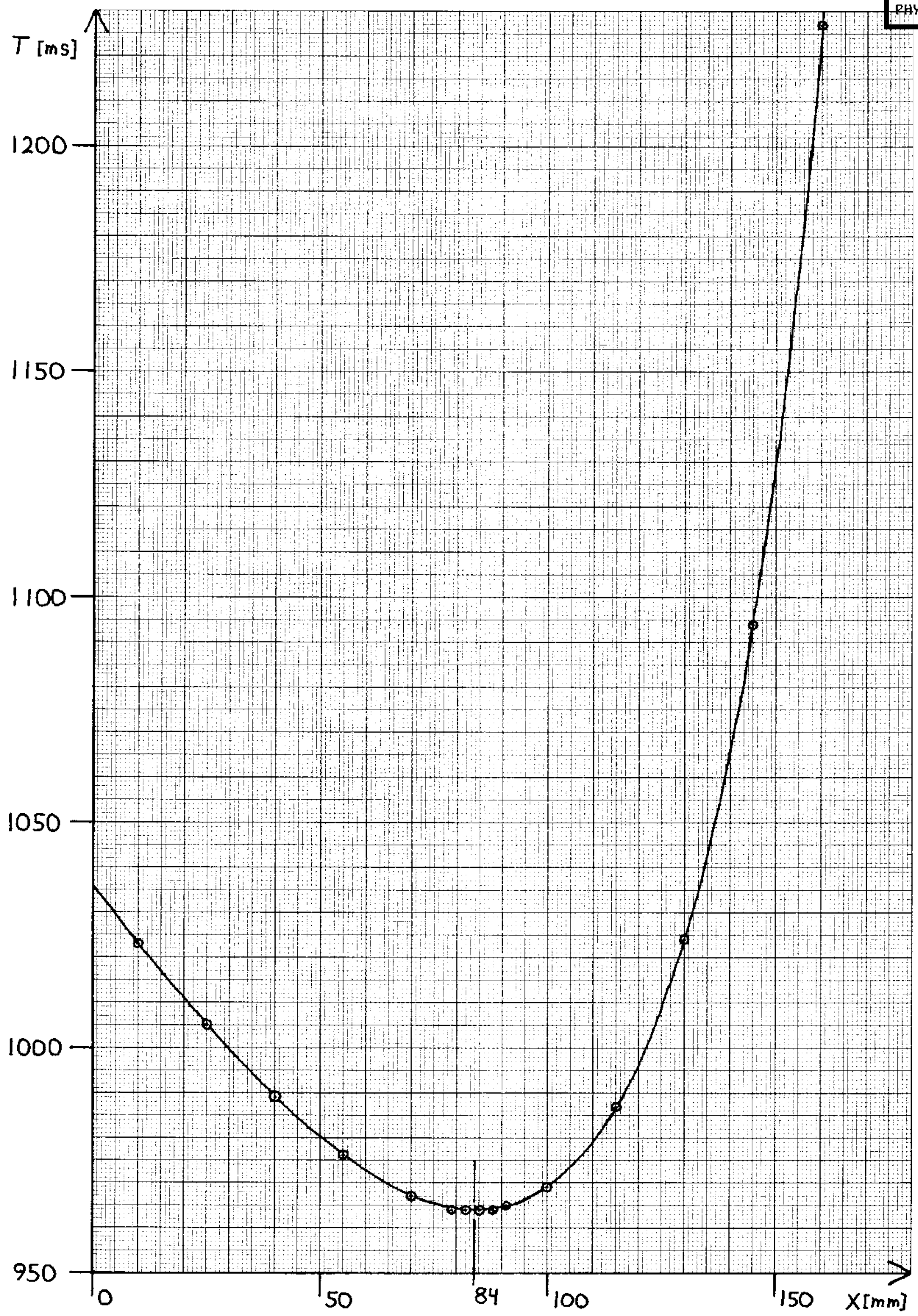
Anticipated INCORRECT answers are printed on grey background and are included to point out places where the students may make mistakes or approximations without being aware of them.

Section 1:

1a) Threads are 1.50 mm/turn. Counted turns to measure position x.

Turn no.	0	10	20	30	40	50	60	70	80	90	100
x [mm]	10.0	25.0	40.0	55.0	70.0	85.0	100.0	115.0	130.0	145.0	160.0
T [ms]	1023	1005	989	976	967	964	969	987	1024	1094	1227

Turn no.	110	120		46	48	52	54
x [mm]	175.0	190.0		79.0	82.0	88.0	91.0
T [ms]	1490	2303		964	964	964	965



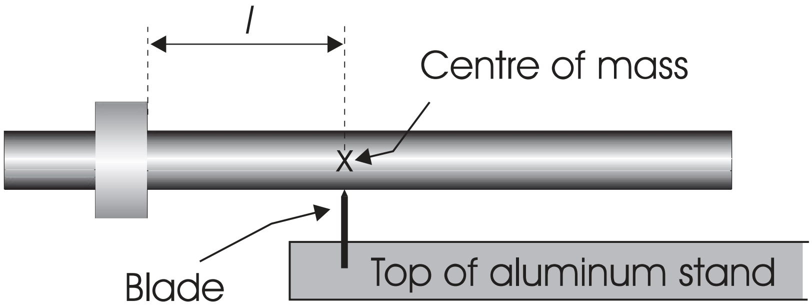
1b) Graph: *T(x)*, shown above.

T = 950 ms: NO positions
T =1000 ms: 2 positions
T =1100 ms: 1 position

If the answer is given as corresponding x-values, and these reflect the number of positions asked for, this answer will also be accepted.

1c) Minimum on graph: x = 84 mm, (estimated uncertainty 1 mm)

By balancing the pendulum horizontally: l = 112.3 mm + 0.55 mm = 113 mm



ALTERNATIVE 1c-1:

$$x_{CM} = \frac{M_{ROD}L - M_{NUT}h}{2M} + \frac{M_{NUT}}{M}x = 197.3 \text{ mm for } x = 84 \text{ mm}$$

gives l = 197.3 mm - 84 mm = 113 mm

$M = M_{ROD} + M_{NUT}$, h = 8.40 mm = height of nut minus two grooves.

INCORRECT 1c-1: Assuming that the centre of mass for the pendulum coincides with the midpoint, L/2, of the rod gives $l = L/2 - x$ = 116 mm.

(The exact position of the minimum on the graph is x = 84.4 mm. with l = 112.8 mm)

Section 2:

2a) $l_2 = \frac{I}{Ml_1} = \frac{2100 \text{ mm}^2}{60 \text{ mm}} = 35 \text{ mm}$

See also Figure 6 on the next page

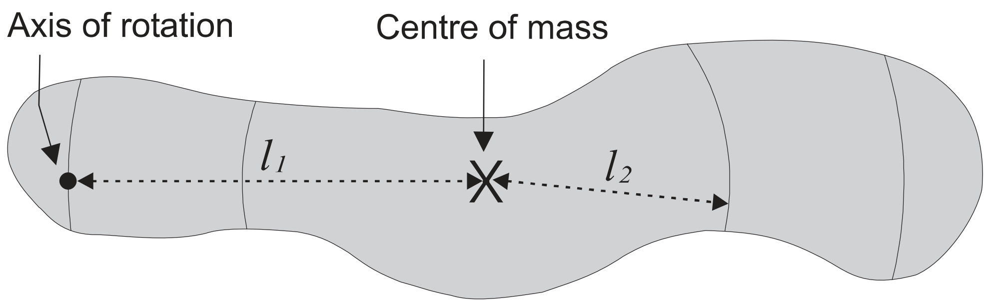


Figure 6. **For use in section 2a**. *Mark* all *positions where a rotation axis (orthogonal to the plane of the paper) can be placed without changing the oscillation period. Assume for this pendulum (drawn on scale, 1:1) that $I/M = 2100\ mm^2$.* (Note: In this booklet the size of this figure is about 75% of the size in the original examination paper.)

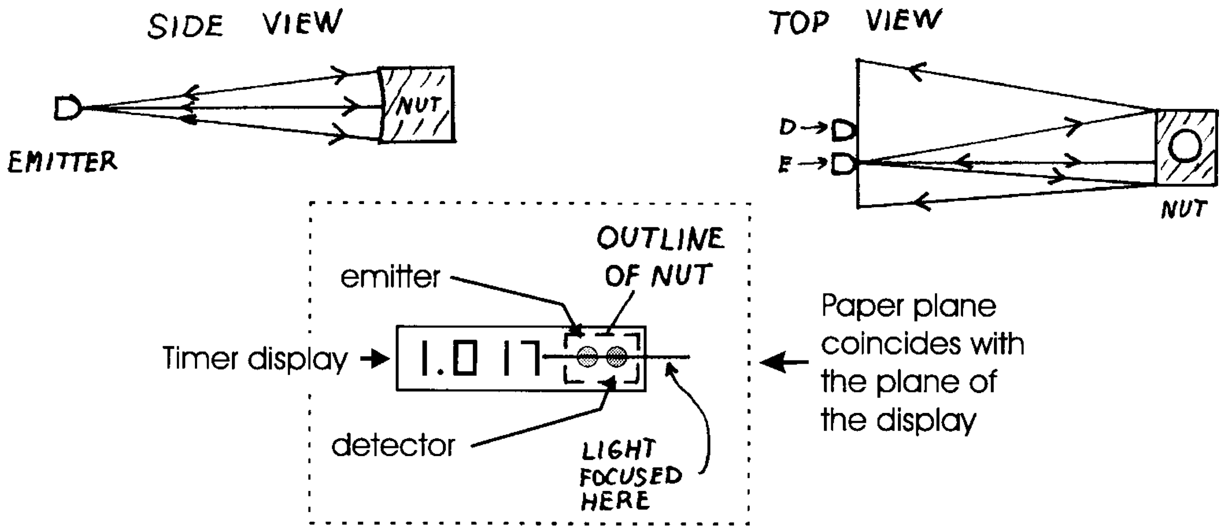


Figure 7. **For use in section 3b.** *Indicate the whole area where the reflected light hits when the pendulum is vertical.*

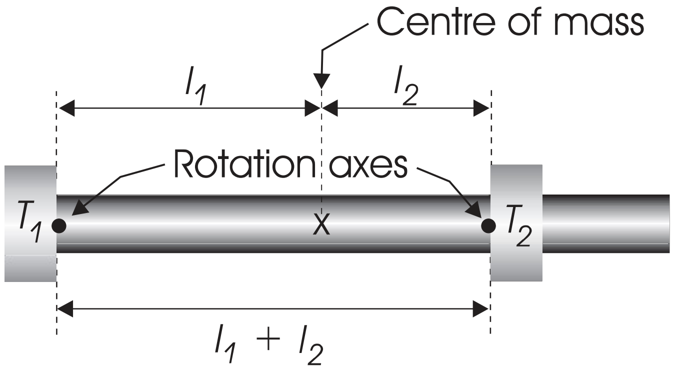
Include this page in your report!

2b) Simple method with small uncertainty: Inverted pendulum.

Equation (1) + (2) $\Rightarrow T_1 = T_2 = \frac{2\pi}{\sqrt{g}}\sqrt{l_1 + l_2} \Leftrightarrow g = \frac{4\pi^2}{T_1^2}(l_1 + l_2)$

NOTE: Independent of I/M !

Used both nuts with one nut at the end to maximise $l_1 + l_2$. Alternately adjusted nut positions until equal periods $T_1 = T_2$:



$T_1 = T_2 = 1024$ ms.

Adding the depth of the two grooves to the measured distance between nuts:

$l_1 + l_2 = (259.6 + 2 \cdot 0.55)$ mm = 0.2607 m

$$g = \frac{4\pi^2}{T_1^2}(l_1 + l_2) = \frac{4 \cdot 3.1416^2 \cdot 0.2607\text{m}}{(1.024\text{s})^2} = \underline{\underline{9.815\text{m}/\text{s}^2}}$$

ALTERNATIVE 2b-1: Finding I(x). *Correct but time consuming.*

It is possible to derive an expression for I as a function of x. By making sensible approximations, this gives:

$$\frac{I(x)}{M} = \left[\frac{L^2}{12} + \frac{M_{NUT}}{M}\left(\frac{L+h}{2} - x\right)^2\right]\frac{M_{ROD}}{M}$$

which is accurate to within 0.03 %. Using the correct expression for l as a function of x:

$$l(x) = x_{CM} - x = \frac{M_{ROD}L - M_{NUT}h}{2M} - \frac{M_{ROD}}{M}x = 195.3 \text{ mm} - 0.9773x,$$

equation (1) can be used on any point (x, T) to find g. Choosing the point (85 mm, 964 ms) *gives:*

$$g = \frac{4\pi^2}{T^2}\left[\frac{I(x)}{M \cdot l(x)} + l(x)\right] = \frac{4 \cdot 3.1416^2 \cdot 0.2311\text{m}}{(0.964\text{s})^2} = \underline{\underline{9.818 \text{ m}/\text{s}^2}}$$

*Using the minimum point on the graph in the way shown below is wrong, since the curve in **1b)**,* $T(x)=\frac{2\pi}{\sqrt{g}}\sqrt{\frac{I(x)}{M\cdot l(x)}+l(x)}$ *with I(x)/M and l(x) given above, describes a continuum of **different** pendulums with changing I(x) and moving centre of mass.*

Equation (1): $T=\frac{2\pi}{\sqrt{g}}\sqrt{\frac{I}{Ml}+l}$ *describes **one** pendulum with fixed I, and does not apply to the curve in **1b)**.*

INCORRECT 2b-1: At the minimum point we have from Equation (2) and **1c)**:

$l_1 = l_2 = l = \sqrt{I/M} = (113 \pm 1)\text{ mm}$ Equation (1) becomes

$$T_{\min} = \frac{2\pi}{\sqrt{g}}\sqrt{\frac{l^2}{l}+l} = \frac{2\pi}{\sqrt{g}}\sqrt{2l}$$ and

$$g = \frac{8\pi^2 l}{T_{\min}^{\ 2}} = \frac{8\cdot 3.1416^2\cdot 0.113\text{ m}}{(0.964\text{ s})^2} = 9.60\text{ m/s}^2$$

Another source of error which may accidentally give a reasonable value is using the wrong value l = (116 ± 1)mm from «INCORRECT 1c-1»:

INCORRECT 2b-2: $g = \frac{8\pi^2 l}{T_{\min}^{\ 2}} = \frac{8\cdot 3.1416^2\cdot 0.116\text{ m}}{(0.964\text{ s})^2} = 9.86\text{ m/s}^2$

Totally neglecting the mass of the nut but remembering the expression for the moment of inertia for a thin rod about a perpendicular axis through the centre of mass, $I = ML^2/12$, *gives from equation (2) for the minimum point:* $l^2 = I/M = L^2/12 = 0.01333\text{ m}^2$. *This value is accidentally only 0.15% smaller than the correct value for I(x)/M at the minimum point on the curve in **1b)**:*

$$\frac{I(x=84.43\text{ mm})}{M} = \left[\frac{L^2}{12}+\frac{M_{NUT}}{M}\left(\frac{L+h}{2}-x\right)^2\right]\frac{M_{ROD}}{M} = 0.01335\text{ m}^2.$$

(continued on next page)

(cont.)

Neglecting the term $\frac{M_{NUT}}{M}\left(\frac{L+h}{2}-84.43\,\text{mm}\right)^2 = 0.00033\ \text{m}^2$ *is nearly compensated by omitting the factor* $\frac{M_{ROD}}{M}$ =0.977. *However, each of these approximations are of the order of 2.5 %, well above the accuracy that can be achieved.*

INCORRECT 2b-3: At the minimum point equation (2) gives $l^2 = \frac{I}{M} = \frac{L^2}{12}$. Then

$$T_{\min} = \frac{2\pi}{\sqrt{g}}\sqrt{2l} = \frac{2\pi}{\sqrt{g}}\sqrt{\frac{2L}{\sqrt{12}}} = \frac{2\pi}{\sqrt{g}}\sqrt{\frac{L}{\sqrt{3}}} \quad \text{and}$$

$$g = \frac{4\pi^2 L}{\sqrt{3}\,T_{\min}^{\ 2}} = \frac{4\cdot 3.1416^2\cdot 0.4000\ \text{m}}{1.7321\cdot(0.964\ \text{s})^2} = 9.81\ \text{m}/\text{s}^2$$

2c) Estimating uncertainty in the logarithmic expression for g:

Let $S \equiv l_1 + l_2 \quad \Rightarrow \quad g = \frac{4\pi^2 S}{T^2}$

$\Delta S = 0.3$ mm $\qquad \Delta T = 1$ ms

$$\frac{\Delta g}{g} = \sqrt{\left(\frac{\Delta S}{S}\right)^2 + \left(-2\frac{\Delta T}{T}\right)^2} = \sqrt{\left(\frac{0.3\ \text{mm}}{260.7\ \text{mm}}\right)^2 + \left(2\cdot\frac{1\ \text{ms}}{1024\ \text{ms}}\right)^2}$$

$$= \sqrt{(0.0012)^2 + (0.0020)^2} = 0.0023 = 0.23\%$$

$$\Delta g = 0.0023\cdot 9.815\ \text{m}/\text{s}^2 = 0.022\ \text{m}/\text{s}^2$$

$$\underline{\underline{g = (9.82 \pm 0.02)\ \text{m}/\text{s}^2}}$$

The incorrect methods INCORRECT 2b-1, 2b-2 and 2b-3 have a similar expressions for g as above. With Δl = 1 mm in INCORRECT 2b-1 and 2b-2 we get Δg = 0.09 m/s^2. INCORRECT 2b-3 should have Δl = 0.3 mm and Δg = 0.02 m/s^2.

ALTERNATIVE 3 has a very complicated x dependence in g. Instead of differentiating g(x) it is easier to insert the two values x+Δx and x-Δx in the expression in brackets [ ], thus finding an estimate for Δ[ ] and then using the same formula as above.

(The official local value for g, measured in the basement of the adjacent building to where the practical exam was held is g = 9.8190178 m/s^2 with uncertainty in the last digit.)

Section 3.

3a) 3. Cylindrical mirror
4. Concave mirror

Radius of curvature of cylinder, r = 145 mm. *(Uncertainty approx. ± 5 mm, not asked for.)*

(In this set-up the emitter and detector are placed at the cylinder axis. The radius of curvature is then the distance between the emitter/detector and the mirror.)

3b) Three drawings, see Figure 7 on page 4 in this Model Answers.

(The key to understanding this set-up is that for a concave cylindrical mirror with a point source at the cylinder axis, the reflected light will be focused back onto the cylinder axis as a line segment of length twice the width of the mirror.)

Section 4.

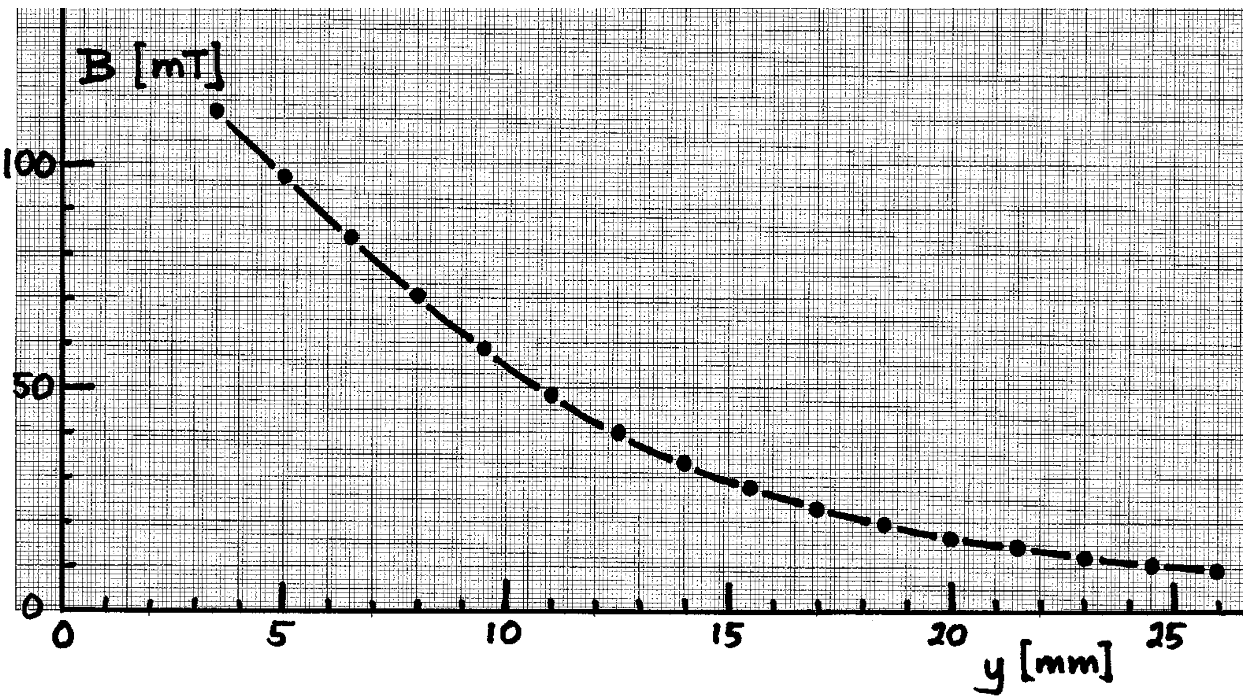
4a) V_o = 2.464 V *(This value may be different for each set-up.)*

4b) Threads are 1.50 mm /turn. Measured *V(y)* for each turn. Calculated

$$B(y) = \left[V(y) - V_0\right]\frac{\Delta B}{\Delta V} = \left[V(y) - V_0\right] / \frac{\Delta V}{\Delta B}.$$ *(Table not requested)*

See graph on next page.

Graph: B(y):



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4c)

$$B_0 = B(y)\left[\frac{y+t}{\sqrt{(y+t)^2+r^2}} - \frac{y}{\sqrt{y^2+r^2}}\right]^{-1}$$

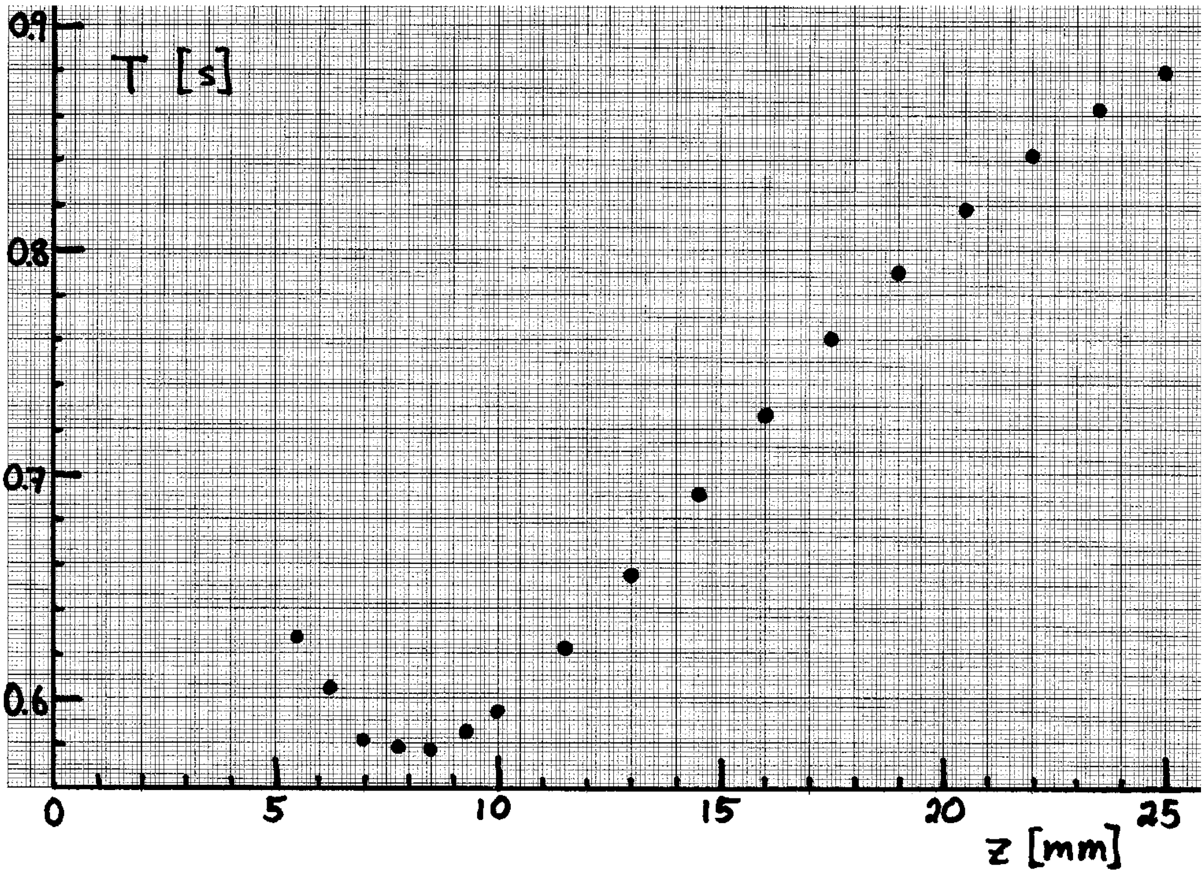
The point (11 mm, 48.5 mT) gives B_0 = 0.621 T and (20 mm, 16,8 mT) gives B_0 = 0.601 T. Mean value: B_0 = 0.61 T *(This value may vary for different magnets.)*

Section 5:

5a) Used the spacer and measured *T(z)* from *z* = 25 mm to 5.5 mm. *(Table is not requested.)*

See plot on next page.

Graph: T(z):



5b) $l(x = 100 \text{ mm}) = 97.6$ mm *(by balancing the pendulum or by calculation as in* ***1c)****.*

$$M = M_{ROD} + M_{NUT}$$

Proportionality means: $\frac{1}{T^2} = a\left[1 + \frac{\mu B_0}{Mgl} f(z)\right]$ where a is a proportionality constant. Setting $B_0 = 0$ corresponds to having an infinitely weak magnet or no magnet at all. Removing the large magnet gives: $T_0 = 968$ ms and $\frac{1}{T_0{}^2} = a\left[1 + 0 \cdot \frac{\mu}{Mgl} f(z)\right]$ or $a = \frac{1}{T_0{}^2}$.

Selecting the point where *f(z)*, see Fig. 5, changes the least with z, i.e., at the maximum, one has $f_{max} = 56.3$. This point must correspond to the minimum oscillation period, which is measured to be $T_{min} = 576$ ms.

We will often need the factor

$$\frac{Mgl}{B_0} = \frac{0.215 \text{ kg} \cdot 9.82 \text{ m/s}^2 \cdot 0.0976 \text{ m}}{0.61 \text{ T}} = 0.338 \text{ Am}^2.$$

.

The magnetic moment then becomes

$$\mu = \frac{Mgl}{B_0}\frac{1}{f_{max}}\left[\left(\frac{T_0}{T}\right)^2 - 1\right] = \frac{0.338\ \text{Am}^2}{56.3}\left[\left(\frac{968}{576}\right)^2 - 1\right] = \underline{\underline{1.1\cdot 10^{-2}\ \text{Am}^2}}$$

ALTERNATIVE 5b-1: *Not what is asked for:* Using *two* points to eliminate the proportionality constant *a*: Equation (4) or $\frac{1}{T^2} = a\left[1 + \frac{\mu B_0}{Mgl} f(z)\right]$ gives:

$$aT_1^2\left[1 + \frac{\mu B_0}{Mgl} f(z_1)\right] = aT_2^2\left[1 + \frac{\mu B_0}{Mgl} f(z_2)\right]$$

$$T_1^2 + T_1^2 \frac{\mu B_0}{Mgl} f(z_1) = T_2^2 + T_2^2 \frac{\mu B_0}{Mgl} f(z_2)$$

$$\frac{\mu B_0}{Mgl}\left[T_1^2 f(z_1) - T_2^2 f(z_2)\right] = T_2^2 - T_1^2$$

$$\mu = \frac{Mgl}{B_0}\cdot\frac{T_2^2 - T_1^2}{T_1^2 f(z_1) - T_2^2 f(z_2)}$$

Choosing two points (z_1 = 7 mm, T_1 = 580.5 ms) and (z_2 = 22 mm, T_2 = 841ms). Reading from the graph $f(z_1)$ = 56.0 and $f(z_2)$ = 12.0 we get

$$\mu = 0.338\ \text{Am}^2 \cdot \frac{841^2 - 580^2}{580^2\cdot 56.0 - 841^2\cdot 12.0} = \underline{1.2\cdot 10^{-2}\ \text{Am}^2}$$

Candidate:	**Total score:** + + + + =
Country:	Marker's name:
Language:	Comment:

Marking Form

for the *Experimental Competition* at the 27th International Physics Olympiad Oslo, Norway

July 4, 1996

To the marker: *Carefully read through the candidate's exam papers and compare with the model answer. You may use the transparencies (handed out) when checking the graph in* ***1b)*** *and the drawing in* ***2a)****. When encountering words or sentences that require translation, postpone marking of this part until you have consulted the interpreter.*

Use the table below and mark a circle around the point values to be subtracted. Add vertically the points for each subsection and calculate the score.
NB: Give score 0 if the value comes out negative for any subsection.
Add the scores for each subsection and write the sum in the 'Total score'- box at the upper right. Keep decimals all the way.

If you have questions, consult the marking leader. Good luck, and remember that you will have to defend your marking in front of the team leaders.

(Note: The terms "INCORRECT 2b-1" found in the table for subsection 2c) and similar terms elsewhere, refer to the Model Answer, in which anticipated incorrect answers were included and numbered for easy reference.)

Subsection 1a) Deficiency	*Subtract*
No answer	**1.0**
x lacks unit	0.1
Other than 0 or 1 decimal in x	0.1
x does not cover the interval 10 mm - 160 mm	0.1
T lacks unit	0.1
T given with other than 1 or 0.5 millisecond accuracy	0.1
Fewer than 11 measuring points (15 mm sep.). Subtr. up to	0.2
Systematic error in x (e.g. if measured from the top of the nut so that the first x = 0 mm)	0.2
If not aware of doubling of the timer period	0.2
Other (specify):	
Score for subsection 1a: **1.0 -**	=

Subsection 1b) Deficiency	*Subtract*
No answer	**1.0**
Lacks "x [(m)m]" on horizontal axis	0.1
1 mm on paper does not correspond to 1 mm in x	0.1
Fewer than 3 numbers on horizontal axis	0.1
Lacks "T [(m)s]" on vertical axis	0.1
1 mm on paper does not correspond to 1 ms in T	0.1
Fewer than 3 numbers on vertical axis	0.1
Measuring points not clearly shown (as circles or crosses)	0.2
More than 5 ms deviation in more than 2 measuring points on the graph	0.2
Wrong answer to the questions (x-values give full score if correct number of values: 0, 2, 1)	0.2
Other (specify):	
Score for subsection 1b): **1.0 -**	=

Subsection 1c) Deficiency	*Subtract*
No answer	**2.0**
x outside the interval 81 - 87 mm. Subtract up to	0.4
x lacks unit	0.1
x given more (or less) accurately than in whole millimeters	0.3
l lacks unit	0.1
l given more (or less) accurately than the nearest mm	0.3
Wrong formula (e.g. $l = 200.0$ mm - x) or something other than $l = x_{CM} - x$	0.6
If it is not possible to see which method was used to find the center of mass	0.2
Other (specify):	
Score for subsection 1c): **2.0 -**	=

Subsection 2a)

	Deficiency	*Subtract*
	No answer	**1.5**
	If drawn straight (vertical) lines	0.4
	If **points** are drawn	0.5
	Other than 4 regions are drawn	0.5
	Inaccurate drawing (> ± 2 mm)	0.3
	Lacks the values l_1 = 60 mm, l_2 = 35 mm on figure or text	0.3
Other (specify):		
	Score for subsection 2a): **1.5 -**	=

Subsection 2b)

	Deficiency	*Subtract*
	No answer	**2.5**
	Lacks (derivation of) formula for g	0.3
	For INVERTED PENDULUM: Lacks figure	0.2
	Values from possible new measurements not given	0.3
	Incomplete calculations	0.3
	If hard to see which method was used	0.4
	Used the formula for INVERTED PENDULUM but read l_1 and l_2 from graph in **1b)** by a horizontal line for a certain T	1.5
	Used one of the other incorrect methods	2.0
	Other than 3 (or 4) significant figures in the answer	0.3
	g lacks unit m/s^2	0.1
Other (specify):		
	Score for subsection 2b): **2.5 -**	=

Subsection 2c)

	Deficiency	*Subtract*
	No answer	**2.5**
	Wrong expression for $\Delta g/g$ or Δg. Subtract up to	0.5
	For INVERTED PENDULUM: If 0.3 mm >$\Delta(l_1+l_2)$ > 0.5 mm	0.2
	For ALTERNATIVE 2c-1: If $\Delta[]$ > 0.1 mm	0.2
	For INCORRECT 2c-1 and 2c-2: If 1 mm > Δl > 2 mm	0.2
	For INCORRECT 2c-3: If 0.3 mm > ΔL > 0.4 mm	0.2
	For all methods: If $\Delta T \neq$ 1 (or 0.5) ms	0.2
	Error in the calculation of Δg	0.2
	Lacks answer including $g \pm \Delta g$ with 2 decimals	0.2
	$g \pm \Delta g$ lacks unit	0.1
Other (specify):		
	Score for subsection 2c): **2.5 -**	=

Subsection 3a)	*Deficiency*	*Subtract*
	No answer	**1.0**
	Lacks point **3. cylindrical mirror**	0.3
	Lacks point **4. concave mirror**	0.3
	Includes other points (1, 2 or 5), subtract per wrong point:	0.3
	Lacks value for radius of curvature	0.4
	If $r < 130$ mm or $r > 160$ mm, subtract up to	0.2
	If r is given more accurately than hole millimeters	0.2
Other (specify):		
	Score for subsection 3a): **1.0 -**	=

Subsection 3b)	*Deficiency*	*Subtract*
	No answer	**2.0**
	Lacks side view figure	0.6
	Errors or deficiencies in the side view figure. Subtract up to	0.4
	Lacks top view figure	0.6
	Errors or deficiencies in the top view figure. Subtract up to	0.4
	Drawing shows light focused to a **point**	0.3
	Drawing shows light spread out over an ill defined or wrongly shaped surface	0.3
	Line/surface is not horizontal	0.2
	Line/point/surface not centered symmetrically on detector	0.2
	Line/point/surface has length different from twice the width of the nut (i.e. outside the interval 10 - 30 mm)	0.1
Other (specify):		
	Score for subsection 3b): **2.0 -**	=

Subsection 4a)	*Deficiency*	*Subtract*
	No answer	**1.0**
	V_o lacks unit V	0.1
	Less than 3 decimals in V_o	0.1
	Incorrect couplings (would give $V_o < 2.3$ V or $V_o > 2.9$ V)	0.8
Other (specify):		
	Score for subsection 4a): **1.0 -**	=

Subsection 4b)	*Deficiency*	*Subtract*
	No answer	**1.5**
	Forgotten V_o or other errors in formula for B	0.2
	Lacks *"y* [(m)m]*"* on horizontal axis	0.1
	Fewer than 3 numbers on horizontal axis	0.1
	Lacks *"B* [(m)T]*"* on vertical axis	0.1
	Fewer than 3 numbers on vertical axis	0.1
	Fewer than 9 measuring points. Subtract up to	0.2
	Measuring points do not cover the interval 3.5 mm - 26 mm	0.2
	Measuring points not clearly shown (as circles or crosses)	0.1
	Error in data or unreasonably large spread in measuring points. Subtract up to	0.5
Other (specify):		
	Score for subsection 4b): **1.5 -**	=

Subsection 4c)	*Deficiency*	*Subtract*
	No answer	**1.5**
	Incorrect formula for B_o	0.3
	If used only one measuring point	0.4
	If used untypical points on the graph	0.3
	Errors in calculation of mean value for B_o	0.2
	B_o lacks unit T	0.1
	Other than two significant figures in (the mean value of) B_o	0.2
	$B_o < 0.4$ T or $B_o > 0.7$ T. Subtract up to	0.2
Other (specify):		
	Score for subsection 4c): **1.5 -**	=

Subsection 5a) Deficiency	Subtract
No answer	**1.0**
Lacks *"z [(m)m]"* on horizontal axis	0.1
Fewer than 3 numbers on horizontal axis	0.1
Lacks *"T [(m)s]"* on vertical axis	0.1
Fewer than 3 numbers on vertical axis	0.1
Fewer than 8 measuring points. Subtract up to	0.2
Measuring points not clearly shown (as circles or crosses)	0.1
Measuring points do not cover the interval 5.5 mm - 25 mm	0.2
Error in data (e.g. plotted $2T$ instead of T) or unreasonably large spread in measuring points. Subtr. up to	0.5
Other (specify):	
Score for subsection 5a): **1.0 -**	=

Subsection 5b) Deficiency	Subtract
No answer	**3.0**
Forgotten center of mass displacement in l (used l = 100 mm)	0.3
Used ALTERNATIVE 5b-1	1.0
Lacks method for finding the proportionality factor a	2.5
Not found correct proportionality factor a	0.3
Used another point than the maximum of $f(z)$	0.1
Incorrect reading of $f(z)$	0.1
Used M_{ROD} or another incorrect value for M	0.2
Incorrect calculation of μ	0.3
μ lacks unit (Am^2 or J/T)	0.2
More than 2 significant figures in μ	0.3
Other (specify):	
Score for subsection 5b): **3.0 -**	=

Total points:

Total for section 1 (max. 4 points):
Total for section 2 (max. 5 points):
Total for section 3 (max. 3 points):
Total for section 4 (max. 4 points):
Total for section 5 (max. 4 points):

The last preparations

The problem for the experimental competition was discussed by the leaders and the organizers the evening before the exam. At this meeting the equipment was demonstrated for the first time (picture).

Photo: Børge Holme

After the meeting had agreed on the final text (in English), the problems had to be translated into the remaining 36 languages. One PC was available for each nation for the translation process (see picture below). The last nation finished their translation at about 4:30 a.m. in the morning, and the competition started at 0830. Busy time for the organizers! Examples of the different translations are given on the following pages.

Photo: Børge Holme