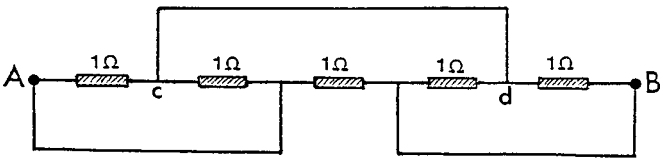
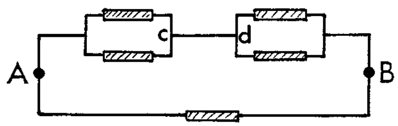
27^{th} INTERNATIONAL PHYSICS OLYMPIAD OSLO, NORWAY

THEORETICAL COMPETITION
JULY 2 1996

Solution Problem 1

a) The system of resistances can be redrawn as shown in the figure:





The equivalent drawing of the circuit shows that the resistance between point c and point A is 0.5Ω, and the same between point d and point B. The resistance between points A and B thus consists of two connections in parallel: the direct 1Ω connection and a connection consisting of two 0.5Ω resistances in series, in other words two parallel 1Ω connections. This yields

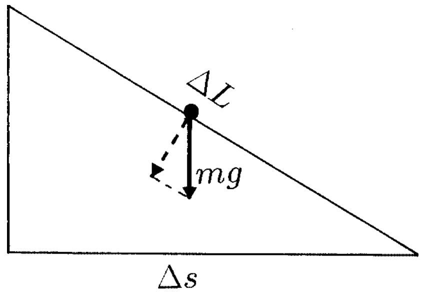
$$R = \underline{\underline{0.5\ \Omega}}\ .$$

b) For a sufficiently short horizontal displacement Δs the path can be considered straight. If the corresponding length of the path element is ΔL, the friction force is given by

$$\mu\, mg\frac{\Delta s}{\Delta L}$$

and the work done by the friction force equals force times displacement:

$$\mu\ mg\frac{\Delta s}{\Delta L}\cdot\Delta L=\mu\ mg\Delta s.$$



Adding up, we find that along the whole path the total work done by friction forces i $\mu\ mg\,s$. By energy conservation this must equal the decrease $mg\,h$ in potential energy of the skier. Hence

$$h=\underline{\underline{\mu s}}.$$

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c) Let the temperature increase in a small time interval dt be dT. During this time interval the metal receives an energy $P\,dt$.

The heat capacity is the ratio between the energy supplied and the temperature increase:

$$C_p=\frac{Pdt}{dT}=\frac{P}{dT/dt}.$$

The experimental results correspond to

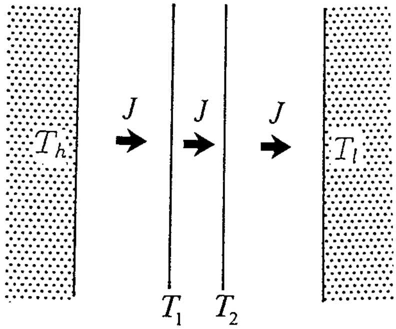
$$\frac{dT}{dt}=\frac{T_0}{4}a[1+a(t-t_0)]^{-3/4}=T_0\frac{a}{4}\left(\frac{T_0}{T}\right)^3.$$

Hence

$$C_p=\frac{P}{dT/dt}=\underline{\frac{4P}{aT_0^{\,4}}T^3}.$$

(*Comment*: At low, but not extremely low, temperatures heat capacities of metals follow such a T^3 law.)

d)



Under stationary conditions the net heat flow is the same everywhere:

$$J = \sigma(T_h^4 - T_1^4)$$
$$J = \sigma(T_1^4 - T_2^4)$$
$$J = \sigma(T_2^4 - T_l^4)$$

Adding these three equations we get

$$3J = \sigma(T_h^4 - T_l^4) = J_0,$$

where J_0 is the heat flow in the absence of the heat shield. Thus $\xi = J/J_0$ takes the value

$$\xi = \underline{\underline{1/3}}.$$

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e) The magnetic field can be determined as the superposition of the fields of two *cylindrical* conductors, since the effects of the currents in the area of intersection cancel. Each of the cylindrical conductors must carry a larger current I', determined so that the fraction I of it is carried by the actual cross section (the moon-shaped area). The ratio between the currents I and I' equals the ratio between the cross section areas:

$$\frac{I}{I'} = \frac{(\frac{\pi}{12} + \frac{\sqrt{3}}{8})D^2}{\frac{\pi}{4} D^2} = \frac{2\pi + 3\sqrt{3}}{6\pi}.$$

Inside one cylindrical conductor carrying a current I' Ampère's law yields at a distance r from the axis an azimuthal field

$$B_\phi = \frac{\mu_0}{2\pi r} \frac{I' \pi r^2}{\frac{\pi}{4} D^2} = \frac{2\mu_0 I' r}{\pi D^2}.$$

The cartesian components of this are

$$B_x = -B_\phi \frac{y}{r} = -\frac{2\mu_0 I' y}{\pi D^2}; \qquad B_y = B_\phi \frac{x}{r} = \frac{2\mu_0 I' x}{\pi D^2}.$$

For the superposed fields, the currents are $\pm I'$ and the corresponding cylinder axes are located at $x = \mp D/4$.

The two x-components add up to zero, while the y-components yield

$$B_y = \frac{2\mu_0}{\pi D^2}[I'(x + D/4) - I'(x - D/4)] = \frac{\mu_0 I'}{\pi D} = \underline{\underline{\frac{6\,\mu_0 I}{(2\pi + 3\sqrt{3})D}}},$$

i.e., a *constant* field. The direction is along the positive y-axis.

Solution Problem 2

a) The potential energy gain eV is converted into kinetic energy. Thus

$$\tfrac{1}{2} mv^2 = eV \qquad \text{(non-relativistically)}$$

$$\frac{mc^2}{\sqrt{1 - v^2/c^2}} - mc^2 = eV \qquad \text{(relativistically).}$$

Hence

$$v = \begin{cases} \sqrt{2eV/m} & \text{(non - relativistically)} \\ c\sqrt{1 - \left(\dfrac{mc^2}{mc^2 + eV}\right)^2} & \text{(relativistically).} \end{cases} \qquad (1)$$

b) When $V = 0$ the electron moves in a homogeneous static magnetic field. The magnetic Lorentz force acts orthogonal to the velocity and the electron will move in a circle. The initial velocity is tangential to the circle.

The radius R of the orbit (the "cyclotron radius") is determined by equating the centripetal force and the Lorentz force: