

## Theoretical Question 2

### Sound Propagation

#### Introduction

The speed of propagation of sound in the ocean varies with depth, temperature and salinity. Figure 1(a) below shows the variation of sound speed  $c$  with depth  $z$  for a case where a minimum speed value  $c_0$  occurs midway between the ocean surface and the sea bed. Note that for convenience  $z = 0$  at the depth of this sound speed minimum,  $z = z_S$  at the surface and  $z = -z_b$  at the sea bed. Above  $z = 0$ ,  $c$  is given by

$$c = c_0 + bz \quad .$$

Below  $z = 0$ ,  $c$  is given by

$$c = c_0 - bz \quad .$$

In each case  $b = \left| \frac{dc}{dz} \right|$ , that is,  $b$  is the magnitude of the sound speed gradient with depth;  $b$  is assumed constant.

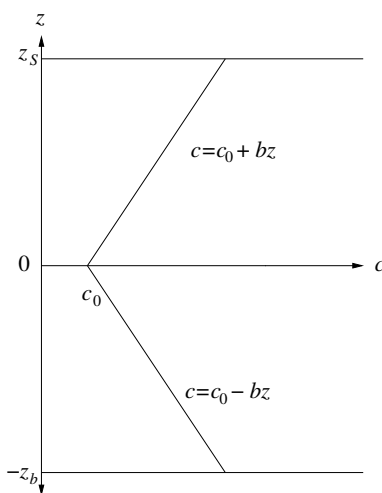


Figure 1 (a)

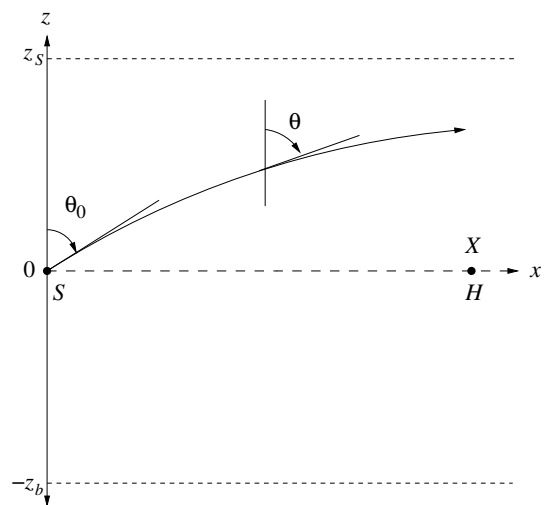


Figure 1 (b)

Figure 1(b) shows a section of the  $z$ - $x$  plane through the ocean, where  $x$  is a horizontal direction. The variation of  $c$  with respect to  $z$  is shown in figure 1(a). At the position  $z = 0$ ,  $x = 0$ , a sound source  $S$  is located. A 'sound ray' is emitted from  $S$  at an angle  $\theta_0$  as shown. Because of the variation of  $c$  with  $z$ , the ray will be refracted.

(a) (6 marks)

Show that the trajectory of the ray, leaving the source  $S$  and constrained to the  $z$ - $x$  plane forms an arc of a circle with radius  $R$  where

$$R = \frac{c_0}{b \sin \theta_0} \quad \text{for } 0 \leq \theta_0 < \frac{\pi}{2} \quad .$$

(b) (3 marks)

Derive an expression involving  $z_S$ ,  $c_0$  and  $b$  to give the smallest value of the angle  $\theta_0$  for upwardly directed rays which can be transmitted without the sound wave reflecting from the sea surface.

(c) (4 marks)

Figure 1(b) shows the position of a sound receiver  $H$  which is located at the position  $z = 0$ ,  $x = X$ . Derive an expression involving  $b$ ,  $X$  and  $c_0$  to give the series of angles  $\theta_0$  required for the sound ray emerging from  $S$  to reach the receiver  $H$ . Assume that  $z_S$  and  $z_b$  are sufficiently large to remove the possibility of reflection from sea surface or sea bed.

(d) (2 marks)

Calculate the smallest four values of  $\theta_0$  for refracted rays from  $S$  to reach  $H$  when

- $X = 10000$  m
- $c_0 = 1500$  ms<sup>-1</sup>
- $b = 0.02000$  s<sup>-1</sup>

(e) (5 marks)

Derive an expression to give the time taken for sound to travel from  $S$  to  $H$  following the ray path associated with the **smallest** value of angle  $\theta_0$ , as determined in part (c). Calculate the value of this transit time for the conditions given in part (d). The following result may be of assistance:

$$\int \frac{dx}{\sin x} = \ln \tan \left( \frac{x}{2} \right)$$

Calculate the time taken for the direct ray to travel from  $S$  to  $H$  along  $z = 0$ . Which of the two rays will arrive first, the ray for which  $\theta_0 = \pi/2$ , or the ray with the smallest value of  $\theta_0$  as calculated for part (d)?