Experimental problem 2...... Solution

1. The circuit diagram is shown in Fig. 1

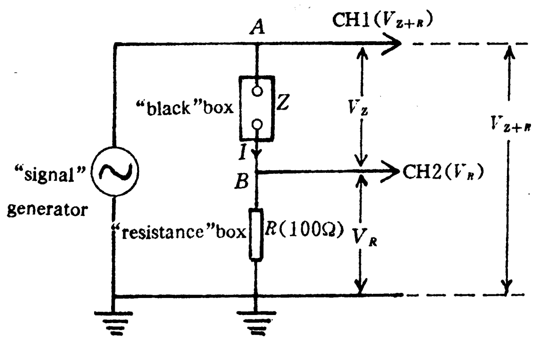

Fig. 1

We have the relation:

$$I = \frac{V_R}{R};$$

$$Z + R = \frac{V_{Z+R}}{I} = \frac{V_{Z+R}}{V_R} R$$

2. Measure the values of V_{Z+R} and V_R at various frequencies (*f*), the measured data and calculated value of *Z*+*R* are shown in table l. "The *Z*+*R*-*f* curve is plotted in Fig. 2

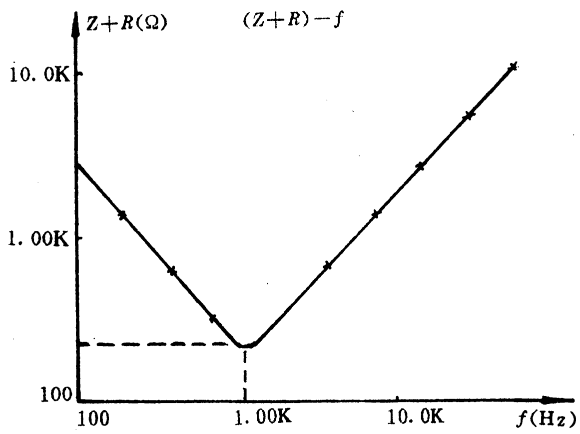

Table l. The magnitude of impedance verus frequency

$f(\times 10^3\ \text{Hz})$	$U_{Z+R}(V_{pp})$	U_R mV$_{pp}$	$Z+R(\times 10^3\,\Omega)$
0.100	0.600	22.0	2.73
0.200	0.600	45.0	1.33
0.400	0.600	94.0	0.638
0.700	0.300	92.0	0.326
0.900	0.300	121	0.248

1.00	0.300	136	0.220
1.10	0.300	140	0.214
1.16	0.300	141	0.213
1.25	0.300	140	0.214
1.50	0.300	120	0.250
2.00	0.300	88.0	0.341
4.00	0.300	78.0	0.769
8.00	0.600	38.0	1.58
15.0	0.600	20.0	3.00
30.0	0.600	10.0	6.00
50.0	0.600	6.0	10.0

From table 1 and Fig. 2, we got the conclusions:

(1) Current resonance (minimum of *Z*) occurs at $f_0 \cong 1.16\times10^3$ Hz.

(2) $f \langle\langle f_0$, $Z \propto f$, $\Delta\varphi \approx -\pi/2$. The impedance of the "black box" at low frequency is dominated by a inductance.

(3) $f \rangle\rangle f_0$, $Z \propto f$, $\Delta\varphi \approx \pi/2$. The impedance of the "black box" at high frequency is dominated by a inductance.

(4) Equivalent circuit of the "black box"; *r*, *L* and *C* connected in series shown in Fig. 3.

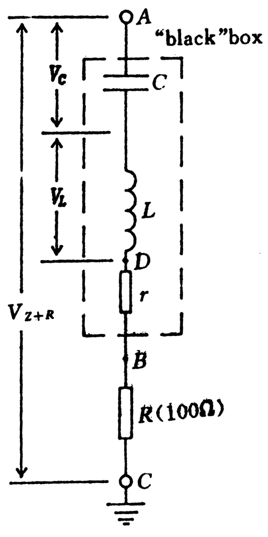

Fig. 3

3. Determination of the values of r, L and C.

(a) r

At resonance frequency f_0

$$V_C = -V_L$$

Then

$$Z + R = \frac{V_{Z+R}}{I} = \frac{V_{Z+R}}{V_R} R = r + R$$

From table 1, $r + R = 213\Omega$, it is given $R = 100\Omega$, so the equivalent resistance r in Fig. 3 is equal $113\,\Omega$.

(b) C

At low frequency, $z_L \approx 0$ in Fig. 3. So the circuit could be considered as a series RC circuit.

From phasor diagram, Fig. 4,

$$\frac{1}{\omega C} = Z_C = \frac{V_C}{I} = \frac{\sqrt{V_{Z+R}^2 - V_{R+r}^2}}{I}$$

Since $V_{R+r}^2 / V_{Z+R}^2 \approx 6\times10^{-3}$ at $f = 100$ Hz, V_{R+r}^2 can beneglected with respect to V_{Z+R}^2, so

$$\frac{1}{\omega C} \approx \frac{V_{Z+R}}{I} \approx Z + R = 2.73\times10^3\,\Omega$$

$$C \approx \frac{1}{\omega(Z+R)} = 0.58\mu f\,.$$

$$C \cong 0.58\mu f\,.$$

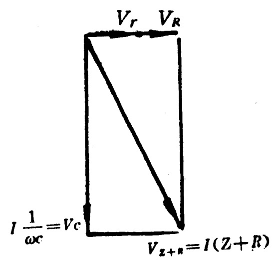

Fig. 4

(c) L

At high frequency, $Z_L \approx 0$ in Fig. 3. So the circuit could be considered as a series RL circuit.

From phasor diagram, Fig. 5,

$$|V_L| = \sqrt{V_{Z+R}^2 - V_{r+R}^2}\,,$$

Since $V_{r+R}^2 / V_{Z+R}^2 \approx 4.5\times10^{-4}$ at $f = 50$ kHz, V_{r+R}^2 can be neglected with respect to V_{Z+R}^2, so

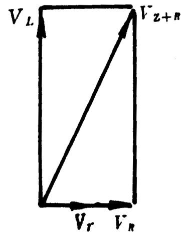

Fig. 5

$$\omega L = Z_L = \frac{V_L}{I} = \frac{|V_{Z+R}|}{I} \approx Z + R = 10^4\,\Omega \tag{3}$$

$$L = \frac{Z+R}{\omega} = 31.8\,\text{mH}.$$

Error estimation:

It is given, precision of the resistance box reading $\Delta R / R \approx 0.5\%$

precision of the voltmeter reading $\Delta V / V \approx 4\%$

(1) Resistance r: at resonance frequency f_0

$$r + R = \frac{V_{Z+R}}{V_R} R$$

$$\frac{\Delta(r+R)}{r+R} = \frac{\Delta V_{Z+R}}{V_{Z+R}} + \frac{\Delta V_R}{V_R} + \frac{\Delta R}{R} \approx 4\% + 4\% + 0.5\% = 8.5\%$$

$$\Delta r = 16\Omega$$

(2) Capacitance C: (Neglect the error of the frequency reading)

$$\frac{1}{\omega C} \cong Z_C = \frac{V_{Z+R}}{V_R} R$$

$$\frac{\Delta C}{C} = \frac{\Delta V_{Z+R}}{V_{Z+R}} + \frac{\Delta V_R}{V_R} + \frac{\Delta R}{R} \approx 8.8\%$$

The approximation $V_C \approx V_{Z+R}$ will introduce apercentageerror 0.3%

(3) Inductance L: Similar to the results of capacitance C, but the percentage error introduced by the approximation $V_L \approx V_{Z+R}$ is much small (0.003%) and thus negligible.

$$\frac{\Delta L}{L} \approx 8.5\%.$$

Experimental Problem 2: Grading Scheme (10 points maximum)

1. Measuring circuit is correct as shown in Fig.(a)

……2.0point

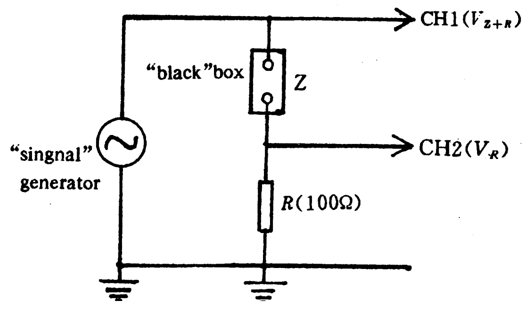

Fig. a

2. Correct data table and figure to show the characteristic of the black box

……2.0 points

3. The equivalent circuit of the black box, and the names of the elements with their values in the equivalent circuit are correct

total 6.0 points

(a) R, L and C are connected in series

……1.5 point

(L and C are connected in series

……1.0 point)

(b) Correct value (error less than 15%) for each element

……0.5 point (×3)

(error between 15% and 30% 0.3)

(error between 30% and 50% 0.1)

(c) Correct calculation formula for each element

……0.5 point (×3)

(d) Error estimate is reasonable for each element

……0.5 points (×3)