**Theoretical Problem 1**

# RELATIVISTIC PARTICLE

In the theory of special relativity the relation between energy $E$ and momentum $P$ or a free particle with rest mass $m_0$ is

$$E = \sqrt{p^2c^2 + m_0^2c^4} = mc^2$$

When such a particle is subject to a conservative force, the total energy of the particle, which is the sum of $\sqrt{p^2c^2 + m_0^2c^4}$ and the potential energy, is conserved. If the energy of the particle is very high, the rest energy of the particle can be ignored (such a particle is called an ultra relativistic particle).

1) consider the one dimensional motion of a very high energy particle (in which rest energy can be neglected) subject to an attractive central force of constant magnitude $f$. Suppose the particle is located at the centre of force with initial momentum $p_0$ at time $t$=0. Describe the motion of the particle by separately plotting, for at least one period of the motion: $x$ against time $t$, and momentum $p$ against space coordinate $x$. Specify the coordinates of the "turning points" in terms of given parameters $p_0$ and $f$. Indicate, with arrows, the direction of the progress of the mothon in the ($p$, $x$) diagram. There may be short intervals of time during which the particle is not ultrarelativistic. However, these should be neglected.

   Use Answer Sheet 1.

2) A meson is a particle made up of two quarks. The rest mass $M$ of the meson is equal to the total energy of the two-quark system divided by $c^2$.

   Consider a one--dimensional model for a meson at rest, in which the two quarks are assumed to move along the $x$-axis and attract each other with a force of constant magnitude f It is assumed they can pass through each other freely. For analysis of the high energy motion of the quarks the rest mass of the quarks can be neglected. At time t=0 the two quarks are both at $x$=0. Show separately the motion of the two quarks graphically by a ($x$, $t$) diagram and a ($p$, $x$) diagram, specify the coordinates of the "turning points" in terms of $M$ and $f$, indicate the direction of the process in your ($p$, $x$) diagram, and determine the maximum distance between the two quarks.

   Use Answer Sheet 2.

3) The reference frame used in part 2 will be referred to as frame $S$, the Lab frame, referred to as $S$, moves in the negative $x$-direction with a constant velocity $v$=0.6$c$. the coordinates in the two reference frames are so chosen that the point

$x$=0 in $S$ coincides with the point $x'=0$ in $S''$ at time $t=t'=0$. Plot the motion of the two quarks graphically in a ($x'$, $t'$) diagram. Specify the coordinates of the turning points in terms of $M$, $f$ and $c$, and determine the maximum distance between the two quarks observed in Lab frame $S'$.

Use Answer Sheet 3.

The coordinates of particle observed in reference frames $S$ and $S''$ are related by the Lorentz transformation

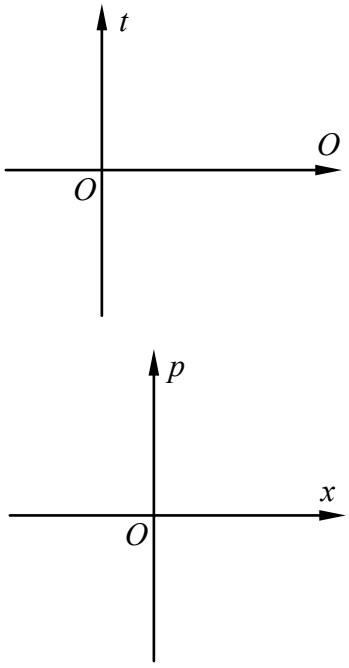
$$\begin{cases} x' = \gamma(x + \beta ct) \\ t' = \gamma(t + \beta \dfrac{x}{c}) \end{cases}$$

where $\beta = v/c$, $\gamma = 1/\sqrt{1-\beta^2}$ and $v$ is the velocity of frame $S$ moving relative to the frame $S''$.

4) For a meson with rest energy $Mc^2$=140 MeV and velocity 0.60$c$ relative to the Lab frame $S''$, determine its energy $E'$ in the Lab Frame $S''$.

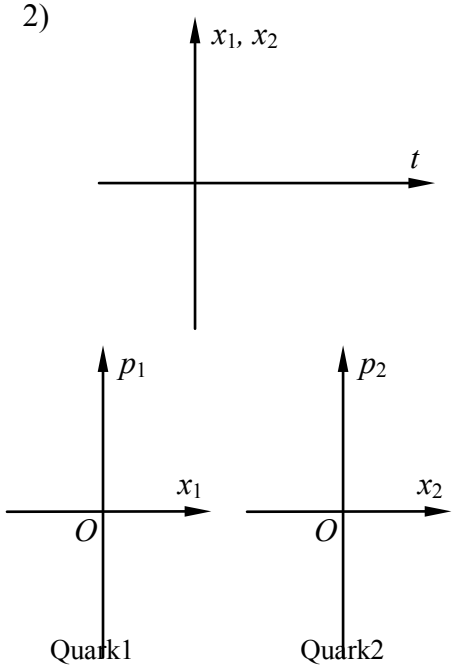
**ANSWER SHEET 1**

1)




**ANSWER SHEET 2**

2)




The maximum distance between the two quarks is $d$=