General Tabulated Information

Quantity	Symbol	Value
Earth's mean radius	R_E	6.4×10^6 m.
acceleration due to gravity	g	9.8 m s^{-2}.
Newtonian gravitational constant	G	6.67×10^{-11} N m^2 kg^{-2}.
permittivity of vacuum	ε_o	8.85×10^{-12} C^2 N^{-1} m^{-2}.
permeability of vacuum	μ_o	8.85×10^{-7} N A^{-2}.
speed of light in vacuum (or air)	c	3.00×10^8 m s^{-1}.
elementary charge	e	1.60×10^{-19} C.
mass of electron	m_e	9.11×10^{-31} kg.
mass of proton	m_p	1.67×10^{-27} kg
Planck constant	h	6.63×10^{-34} J s.
Avogadro constant	N_A	$6,02 \times 10^{23}$ mol^{-1}.
Boltzmann constant	k	1.38×10^{-23} J K^{-1}.
molar gas constant	R	8.31 J mol^{-1} K^{-1}.

ATMOSPHERIC ELECTRICITY

From the standpoint of electrostatics, the surface of the Earth can be considered to be a good conductor. It carries a certain total charge Q_o and an average surface charge density σ_o.

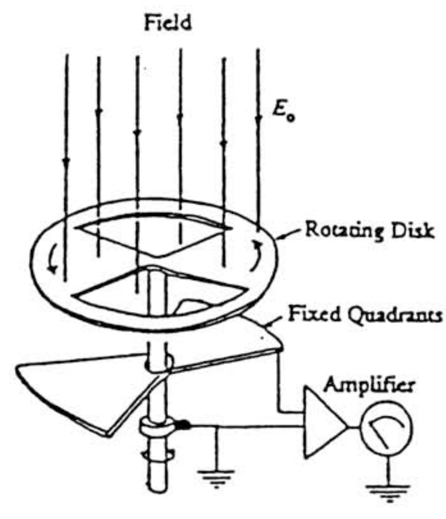
1) Under fair-weather conditions, there is a downward electric field, E_o, at the Earth's surface equal to about 150 V/m. Deduce the magnitude of the Earth's surface charge density and the total charge carried on the Earth's surface.

2) The magnitude of the downward electric field decreases with height, and is about 100 V/m at a height of 100 m. Calculate the average amount of net charge per m^3 of the atmosphere between the Earth's surface and 100 m altitude.

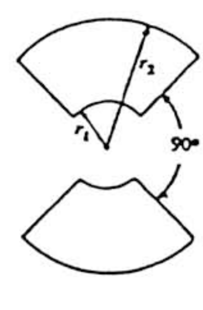
3) The net charge density you have calculated in (2) is actually the result of having almost equal numbers of positive and negative singly-charged ions per unit volume (n_+ and n_-). Near the Earth's surface, under fair-weather conditions, $n_+ \approx n_- \approx 6 \times 10^8\ m^{-3}$. These ions move under the action of the vertical electric field. Their speed is proportional to the field strength:

$$v \approx 1.5 \times 10^{-4} \cdot E,$$

where v is in m/s and E in V/m. How long would it take for the motion of the atmospheric ions to neutralize half of the Earth's surface charge, if no other processes (e.g. lightning) occurred to maintain it?



4) One way of measuring the atmospheric electric field, and hence σ_o, is with the system shown in the diagram. A pair of metal quadrants, insulated from ground but connected to each other, are mounted just underneath a grounded uniformly rotating disk with two quadrant-shaped holes cut in it. (In the diagram, the spacing has been exaggerated in order to show the arrangement.) Twice in each revolution the insulated quadrants are completely exposed to the field, and then (1/4 of a period later) are completely shielded from it. Let T be the period of revolution, and let the inner and outer radii of the insulated quadrants be r_1 and r_2 as shown.



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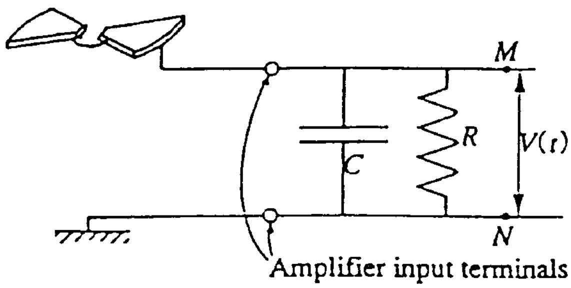
Take $t = 0$ to be an instant when the insulated quadrants are completely shielded.

Obtain expressions that give the total charge $q(t)$ induced on the upper surface of the insulated quadrants as a function of time between $t = 0$ and $t = T/2$, and sketch a graph of this variation.

[The effects of the atmospheric ion current can be ignored in this situation.]

(5) The system described in (4) is connected to an amplifier whose input circuit is equivalent to a capacitor C and a resistor R in parallel. (You can assume that the capacitance of the quadrant system is negligible compared to C.) Sketch graphs of the form of the voltage difference V between the points M and N as a function of t during one revolution of the disk, just after it has been set into rotation with period of revolution T, if:



a) $T = T_a \ll CR$;
b) $T = T_b \gg CR$.

[Assume that C and R have fixed values; only T changes between situations (a) and (b).] Obtain an expression for the approximate ratio, V_a/V_b, of the largest values of $V(t)$ in cases (a) and (b).

6) Assume that $E_0 = 150$ V/m, $r_1 = 1$ cm, $r_2 = 7$ cm, $C = 0.01$ μF, $R = 20$ MΩ, and suppose that the disk is set into rotation at 50 revolutions per second.

Approximately, what is the largest value of V during one revolution in this case?