Problem 3 Cooling Atoms by laser

To study the properties of isolated atoms with a high degree of precision they must be kept almost at rest for a length of time. A method has recently been developed to do this. It is called “laser cooling” and is illustrated by the problem below.

In a vacuum chamber a well collimated beam of Na^{23} atoms (coming from the evaporation of a sample at 10^3 K) is illuminated head-on with a high intensity laser beam (fig. 3.1). The frequency of laser is chosen so there will be resonant absorption of a photon by those atoms whose velocity is v_0. When the light is absorbed, these atoms are exited to the first energy level, which has a mean value E above the ground state and uncertainty of Γ (fig. 3.2).

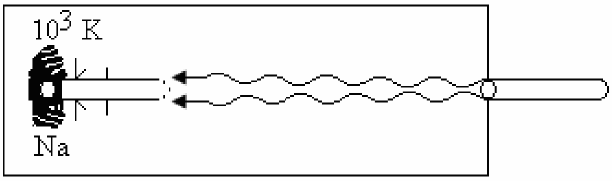


Figure 3.1

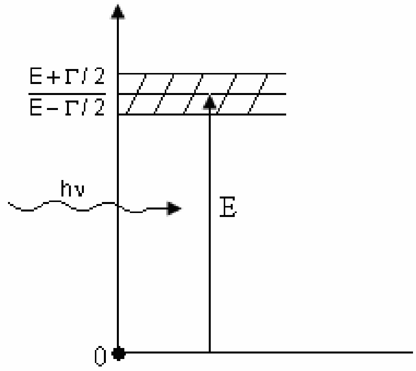


Figure 3.2

For this process, the atom’s decrease in velocity Δv_1 is given by $\Delta v_1 = v_1 - v_0$. Light is then emitted by the atom as it returns to its ground state. The atom’s velocity changes by $\Delta v' = v'_1 - v_1$ and its direction of motion changes by an angle φ (fig. 3.3).

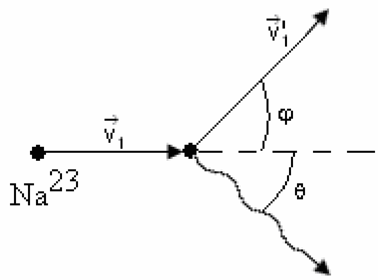


Figure 3.3

This sequence of absorption and emission takes place many times until the velocity of the atoms has decreased by a given amount Δv such that resonant absorption of light at frequency v no longer occurs. It is then necessary to change the frequency of laser so as to maintain resonant absorption. The atoms moving at the new velocity are further slowed down until some of them have a velocity close to zero.

As first approximation we may ignore any atomic interaction processes apart from the spontaneous absorption and emission light described above.
Furthermore, we may assume the laser to be so intense that the atoms spend practically no time in the ground state.

Questions

a) Find the laser frequency needed ensure the resonant absorption of the light by those atoms whose kinetic energy of the atoms inside the region behind the collimator. Also find the reduction in the velocity of these atoms, Δv_1, after the absorption process.

b) Light of the frequency calculated in question a) is absorbed by atoms which velocities lie within a range Δv_0. Calculate this velocity range.

c) When an atom emits light, its direction of motion changes by φ from initial direction. Calculate φ.
d) Find the maximum possible velocity decrease Δv for a given frequency.
e) What is the approximate number N of absorption-emission events necessary to reduce the velocity of an atom from is initial value v_o
-found in question a) above- almost to zero? Assume the atom travels in a straight line.
f) Find the time t that the process in question e takes. Calculate the distance ΔS an atom travels in this time.

Data

$E = 3{,}36 \cdot 10^{-19}$ J
$\Gamma = 7{,}0 \cdot 10^{-27}$ J
$c = 3 \cdot 10^{8}$ ms^{-1}
$m_p = 1{,}67 \cdot 10^{-27}$ kg
$h = 6{,}62 \cdot 10^{-34}$ Js
$k = 1{,}38 \cdot 10^{-23}$ JK^{-1}

where c is speed of light, h is Planck's constant, k is the Boltzmann constant, and mp is the mass of proton.

THEORETICAL PROBLEMS. SOLUTIONS

Solution Problem 1

a) *Calculation of the velocity at the instant before impact*

Equating the potential gravitational energy to the kinetic energy at the instant before impact we can arrive at the pre-impact velocity v_0:

$$mgh = \frac{mv_0^2}{2} \quad (1)$$

from which we may solve for v_0 as follows:

$$V_0 = \sqrt{2gh} \quad (2)$$

b) *Calculation of the vertical component of the velocity at the instant after impact*

Let v_{2x} and v_{2y} be the horizontal and vertical components, respectively, of the velocity of the mass center an instant after impact. The height attained in the vertical direction will be αh and then:

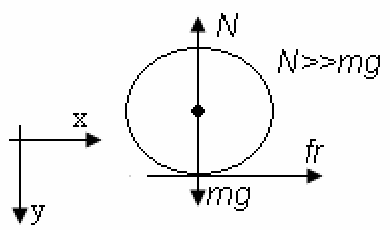
$$v_{2y}^2 = 2g\alpha h \quad (3)$$

from which, in terms of α (or the restitution coefficient $c = \sqrt{\alpha}$):

$$v_{2y} = \sqrt{2g\alpha h} = cv_0 \quad (4)$$

c) *General equations for the variations of linear and angular momenta in the time interval of the Impact*

Figure 1.2 shows the free body of the ball during impact



Considering that the linear impulse of the forces is equal to the variation of the linear momentum and that the angular impulse of the torques is equal to the variation of the angular momentum, we have:

$$I_y = \int_{t_1}^{t_2} N(t)dt = mv_0 + mv_{2y} = m(1+c)\sqrt{2gh} \quad (5)$$

$$I_x = \int_{t_1}^{t_2} f_r(t)dt = mv_{2x} \quad (6)$$