Solution of question 5.

1. *Theory* Let
   - the moment of inertia of the disk be : I
   - the mass of the weight : m
   - the moment of the frictional force : M_f
   - magnetic field strength : B
   - the radius of the axle : r
   - the moment of the magnetic force : M_B

For the motion of the rotating disk we have:

$$I.\alpha = (m.g - m.a).r - M_f - M_B$$

We suppose that M_f is constant but not negligible. Because the disk moves in the magnetic field, eddy currents are set up in the disk. The magnitude of these currents is proportional to B and to the angular velocity. The Lorentz force as a result of the eddy currents and the magnetic field is thus proportional to the square of B and to the angular velocity, i.e.

$$M_B = c.B^2.\omega$$

Substituting this into Eq. (1), we find:

$$I.\alpha = (m.g - m.a).r - M_f - c.B^2.\omega$$

$$v_e = (\frac{g.r^2}{c.B^2}).\left(m - \frac{M_f}{g.r}\right)$$

After some time, the disk will reach its final constant angular velocity; the angular acceleration is now zero and for the final velocity v_e we find:

The final constant velocity is thus a linear function of m.

2. *The experiment*

The final constant speed is determined by measuring the time taken to fall the last 21 cm [this is the width of a sheet of paper].

In the first place it is necessary to check that the final speed has been reached. This is done by allowing the weight to fall over different heights. It is clear that, with the weaker magnet, the necessary height before the constant speed is attained will be larger. Measurements for the weak magnet system:

----------------- time taken to fall -----------------

height (m)	smaller weight	larger weight
0.30	5.04 ± 0.02 (s)	2.00 ± 0.01 (s)
0.40	4.67 ± 0.04 (s)	1.71 ± 0.02 (s)
0.50	4.59 ± 0.05 (s)	1.55 ± 0.02 (s)
0.60	4.44 ± 0.06 (s)	1.48 ± 0.01 (s)
0.70	4.49 ± 0.05 (s)	1.44 ± 0.04 (s)
0.80	4.43 ± 0.03 (s)	1.38 ± 0.03 (s)
0.90	4.43 ± 0.04 (s)	1.35 ± 0.02 (s)
1.10	- - -	1.34 ± 0.05 (s)
1.30	- - -	1.33 ± 0.04 (s)

3. *Final constant speed measurements for both magnet systems and for several choices of weight.*

Measurements for the weak magnet:

weight	T (s)	T (s)	T (s)	T (s)	<T> (s)	<v> (m/s)
small	4.42	4.23	4.24	4.33	4.31 ± 0.09	4.9 ± 0.1
large	1.89	1.91	1.98	1.92	1.93 ± 0.04	10.9 ± 0.2
both	1.29	1.32	1.23	1.30	1.29 ± 0.04	16.3 ± 0.5

Measurements for the strong magnet:

weight	T (s)	T (s)	T (s)	T (s)	<T> (s)	<v> (m/s)
small	8.93	9.01	9.17	8.91	9.0 ± 0.1	2.33 ± 0.03
large	4.03	3.92	4.03	3.95	3.98 ± 0.06	5.28 ± 0.08
both	2.53	2.52	2.53	2.48	2.52 ± 0.03	8.3 ± 0.1

4. *Discussion of the results:*

- A graph between v_e and the weight should be made.
- From Eq. (2) we observe that:
  - both straight lines should intersect on the horizontal axis.
  - from the square-root of the ratio of the slopes we have immediately the ratio of the magnetic field strengths.
  - For the above measurements we find:

$$\frac{B_1}{B_2} = \sqrt{\frac{7.22}{15}} \approx \mathbf{0.69} \quad \rightarrow \quad \frac{\Delta\left(\frac{B_1}{B_2}\right)}{\left(\frac{B_1}{B_2}\right)} = \frac{1}{2}.\sqrt{\left(\frac{\Delta r_1}{r_1}\right)^2 + \left(\frac{\Delta r_2}{r_2}\right)^2} \approx \mathbf{0.05}$$

$$\frac{B_1}{B_2} = \mathbf{0.69 \pm 0.03}$$

Marking Breakdown

1	$M_B = c.B^2.\omega$		: 1
	Eq. (2)		: 1
2	Investigation of the range in which the speed is constant		: 2
3	Number of timing measurements [1,2,3,...]		: 0,1,2
		Error estimation	: 0.5
4	graph	- quality	: 0.5
		- the lines intersect each other on the mass-axis	: 1
		- calculation of B_1/B_2	: 1
		- Error calculation	: 1