Solution of question 4.

1. *The linearity of the photo-diode.*

The linearity of the photo-diode can be checked by using the inverse square law between distance and intensity. Suppose that the measured distance between the LED and the (box containing the) PD is x. The intensity of the light falling on the PD satisfies:

$$I(x) = \frac{I_0}{x^2}$$

If the intensity is indeed proportional to the current flowing through the PD, it will also be proportional to the voltage, V(x), measured across the resistor R3. From (1) it then follows that:

$$\frac{1}{\sqrt{V(x)}} \propto x$$

To obtain the correct value of V(x), one should subtract from the measured voltage V_1 the voltage V_2 that one measures when the LED is turned off (but the LED box is still in place in front of the PD).

x (cm)	V_1 (V)	V_2 (V)	i_1 (μA)	i_2 (μA)	$1/[i_1(x) - i_2(x)]^{1/2}$ ($\mu A^{-1/2}$)
1.0	5.66	.003	6.23	.003	0.40
2.0	4.07	.004	4.48	.005	0.47
3.0	3.03	.005	3.33	.005	0.55
4.0	2.32	.006	2.55	.006	0.63
5.0	1.83	.006	2.01	.006	0.71
6.0	1.48	.007	1.63	.007	0.79
7.0	1.23	.007	1.35	.007	0.86
8.0	1.006	.008	1.107	.008	0.95
9.0	0.859	.009	0.945	.009	1.03
10.0	0.744	.009	0.818	.009	1.11
11.0	0.648	.010	0.713	.010	1.19
12.0	0.570	.011	0.627	.011	1.27
13.0	0.507	.012	0.558	.012	1.35
14.0	0.456	.012	0.502	.012	1.43
15.0	0.414	.013	0.455	.013	1.50
16.0	0.373	.013	0.410	.014	1.59
17.0	0.341	.014	0.375	.014	1.66
18.0	0.312	.014	0.343	.014	1.74
19.0	0.291	.015	0.320	.015	1.81
20.0	0.272	.015	0.299	.015	1.88

Plotted on a graph, one finds a perfect straight line.

2. *The light intensity as a function of the electrical power of the LED*

The photo-current i_{PD} is determined from the voltage V over R3 = 1MΩ. The meter itself has an internal resistance of 100 MΩ in the 200 mV range and 10 MΩ in the other ranges. We have then: $i_{PD} = 1.01$ V resp. $i_{PD} = 1.1$ V where V is in volts and i_{PD} in μA. The current in ampères through the LED is the voltage over R1 in volts, divided by 100.

[x = 5 cm]

PD			LED			
V_1 (V)	V_2(V)	$i_1 - i_2$ (μA)	i_{LED} (10^{-2} A)	V_{LED} (V)	P_{LED} (10^{-2} W)	$(i_1 - i_2)/P_{LED}$
1.806	.0061	1.98	2.70	1.752	4.73	0.419
1.637	.0061	1.79	2.30	1.742	4.01	0.446
1.511	.0061	1.66	2.08	1.735	3.61	0.460
1.225	.0061	1.34	1.606	1.722	2.77	0.484
1.117	.0061	1.22	1.433	1.718	2.46	0.496
0.903	.0061	0.99	1.123	1.705	1.91	0.518
0.711	.0061	0.78	0.889	1.708	1.52	0.513
0.448	.0061	0.49	0.555	1.673	0.93	0.527
0.315	.0061	0.34	0.410	1.659	0.68	0.5
0.192	.0061	0.21	0.258	1.637	0.42	0.2

The efficiency is proportional to $(i_1 - i_2)/P_{LED}$. In the graph of $(i_1 - i_2)/P_{LED}$ against i_{LED} the maximal efficiency corresponds to $i_{LED} = 0{,}6 \cdot 10^{-2}$ A. (See figure 2.)

3. *Determination of the maximal efficiency.*

The LED emits a conical beam with cylindrical symmetry. Suppose we measure the light intensity with a PD of sensitive area d^2 at a distance r_i from the axis of symmetry. Let the intensity of the light there be $\Phi(r_i)$, then we have:

$$i(r_i) = N_e.e = N_f q_f e = \frac{\Phi(r_i)}{h.\nu}.q_f e$$

$$\Phi = \sum_i \Phi(r_i).\frac{2.\pi.r_i.d}{d^2} = \frac{2.\pi}{d}.\sum_i \Phi(r_i).r_i = \frac{2.\pi}{d}.\frac{h.\nu}{q_f e}.\sum_i i(r_i).r_i$$

r_i (mm)	V_1 (V)	V_2 (V)	$(i_1 - i_2).r_i$ (x 10^{-9} Am)	r_i (mm)	V_1 (V)	V_2 (V)	$(i_1 - i_2).r_i$ (x 10^{-9} Am)
0	1.833	0.006	0	39	0.097	0.006	
3	1.906	0.006	6.27	42	0.089	0.006	4.16
6	1.846	0.006	12.54	45	0.082	0.006	3.86
9	1.750	0.006	17.28	48	0.071	0.006	3.79
12	1.347	0.006	17.76	51	0.066	0.006	3.48
15	0.997	0.006	16.20	54	0.050	0.006	3.39
18	0.643	0.006	12.60	57	0.045	0.006	2.52
21	0.313	0.006	7.14	60	0.037	0.006	2.45
24	0.343	0.006	8.88	63	0.032	0.006	2.08
27	0.637	0.006	18.90	66	0.023	0.006	1.83
30	0.681	0.006	22.20	69	0.017	0.006	1.27
33	0.266	0.006	9.57	72	0.014	0.006	0.88
36	0.119	0.006	4.48	75	0.011	0.006	0.68
							0.49

The efficiency = $\Phi/P_{LED} \approx 0.001$

Marking breakdown

1 linearity of the PD
- inverse square law :1.5
- number of measuring points [1,3>; [3,5>; [5,..> :0.5/1.0/1.5
- dark current :0.5
- correct graph :1

2 determination of current at maximal effiency
- principle :0.5
- number of measuring points [1,3>; [3,5>; [5,..> :0.5/1.0/1.5
- graph efficiency-current :0.5
- determination of current at maximal efficiency :0.5

3 determination of the maximal efficiency
- determination of the emitted light intensity :1.5
  - via estimation of the cone cross-section :0.5
  - via measurement of the intensity distribution :1.5
- determination of the maximum efficiency :1