Solution of question 1.

a - Consider first the x-direction. If waves coming from neighbouring slits (with separation d_1) traverse paths of lengths that differ by:

$$\Delta_1 = n_1.\lambda$$

where n_1 is an integer, then a principal maximum occurs. The position on the screen (in the x-direction) is:

$$x_{n_1} = \frac{n_1.\lambda.L}{d_1}$$

since $d_1 << d_2$.

The path difference between the middle slit and one of the slits at the edge is then:

$$\Delta_{(\frac{N_1}{2})} = \frac{N_1}{2}.n_1.\lambda$$

If on the other hand this path difference is:

$$\Delta_{(\frac{N_1}{2})} = \frac{N_1}{2}.n_1.\lambda + \frac{\lambda}{2}$$

then the first minimum, next to the principal maximum, occurs. The position of this minimum on the screen is given by:

$$x_{n_1} + \Delta x = \frac{\left(\frac{N_1}{2}.n_1.\lambda + \frac{\lambda}{2}\right).L}{\frac{N_1}{2}.d_1} = \frac{n_1.\lambda.L}{d_1} + \frac{\lambda.L}{N_1.d_1}$$

$$\rightarrow \quad \Delta x = \frac{\lambda.L}{N_1.d_1}$$

The width of the principal maximum is accordingly:

$$2.\Delta x = 2.\frac{\lambda.L}{N_1.d_1}$$

A similar treatment can be made for the y-direction, in which there are N_2 slits with separation d_2. The positions and widths of the principal maximal are:

$$(x_{n_1},y_{n_2}) = (\frac{n_1.\lambda.L}{d_1},\frac{n_2.\lambda.L}{d_2})$$

$$2.\Delta x = 2.\frac{\lambda.L}{N_1.d_1} \quad ; \qquad 2.\Delta y = 2.\frac{\lambda.L}{N_2.d_2}$$

An alternative method of solution is to calculate the intensity for the 2-dimensional grid as a function of the angle that the beam makes with the screen.

b - In the x-direction the beam 'sees' a grid with spacing a, so that in this direction we have:

$$x_{n_1} = \frac{n_1.\lambda.L}{a} \qquad \qquad \Delta x = 2.\frac{\lambda.L}{N_0.a}$$

In the y-direction, the beam 'sees' a grid with effective spacing a.cos(Θ). Analogously, we obtain:

$$y_{n_2} = \frac{n_2.\lambda.L}{a.\cos(\theta)} \qquad \Delta y = 2.\frac{\lambda.L}{N_0.a.\cos(\theta)}$$

In the z-direction, the beam 'sees' a grid with effective spacing a.sin(Θ). This gives rise to principal maxima with position and width:

$$y'_{n_3} = \frac{n_3.\lambda.L}{a.\sin(\theta)} \qquad \Delta y' = 2.\frac{\lambda.L}{N_1.a.\sin(\theta)}$$

This pattern is superimposed on the previous one. Since sin(Θ) is very small, only the zeroth-order pattern will be seen, and it is very broad, since $N_1.\sin(\Theta) << N_0$. The diffraction pattern from a plane wave falling on a thin plate of a cubic crystal, at a small angle of incidence to the normal, will be almost identical to that from a two-dimensional grid.

c - In Bragg reflection, the path difference for constructive interference between neighbouring planes:

$$\Delta = 2.a.\sin(\phi) \approx 2.a.\phi = n.\lambda \rightarrow \frac{x}{L} \approx 2.\phi \approx \frac{n.\lambda}{a} \rightarrow x \approx \frac{n.\lambda.L}{a}$$

Here ϕ is the angle of diffraction.
This is the same condition for a maximum as in section b.

d - For the distance, √2.a, between neighbouring K ions we have:

$$tg(2\phi) = \frac{x}{L} = \frac{0{,}053}{0{,}1} \approx 0{,}53 \rightarrow a = \frac{\lambda}{2.\sin(\phi)} \approx \frac{0{,}15.10^{-9}}{2\ .\ 0{,}24} \approx 0{,}31\ nm$$

$$K-K \approx \sqrt{2}.0{,}31 \approx 0{,}44\ nm$$

Marking Breakdown

a	position of principal maxima	:1
	width of principal maxima	:3
b	lattice constants	:1
	effect of thickness	:2
c	Bragg reflection	:2
d	Calculation of K-K spacing	:1