***Solution (draft)*[1]**

A. As the holders do not affect vibrations of the disc we may expect antinodes on the flat surfaces of the discs (Fig. 4.2; geometric proportions not conserved). One of the frequencies is expected for

$$l = \tfrac{1}{2}\lambda = \frac{v}{2f},$$

where v denotes the velocity of longitudinal ultrasonic wave (its value is given in the text of the problem), f - the frequency and l - the thickness of the disc. Thus:

$$f = \frac{v}{2l}.$$

Numerically $f = 2 \cdot 10^5$ Hz = 200 kHz.

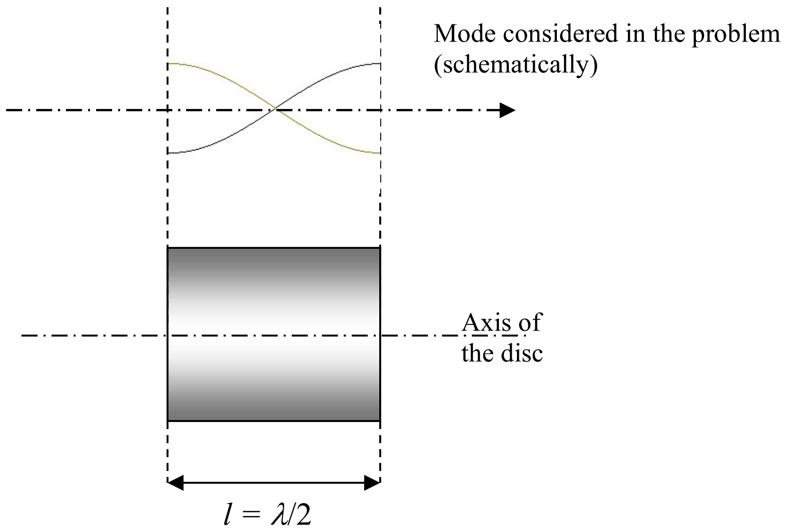

Fig. 4.2

One should stress out that different modes of vibrations can be excited in the disc with height comparable to its diameter. We confine our considerations to the modes related to longitudinal waves moving along the axis of the disc as the sound waves in liquids are longitudinal. We neglect coupling between different modes and require antinodes exactly at the flat parts of the disc. We assume also that the piezoelectric effect does not affect velocity of ultrasound. For these reasons the frequency just determined should be treated as only a rough approximation. However, it indicates that one should look for the resonance in vicinity of 200 kHz.

The experimental set-up is shown in Fig. 4.3. The oscillator (generator) is connected to one of the discs that works as a transmitter and to one channel of the oscilloscope. The second disc is connected to the second channel of the oscilloscope and works as a receiver. Both discs are placed against one of the bags with liquid (Fig. 4.4). The distance d can be varied.

[1] This draft solution is based on the camera-ready text of the more detailed solution prepared by Dr. Andrzej Kotlicki and published in the proceedings [3]

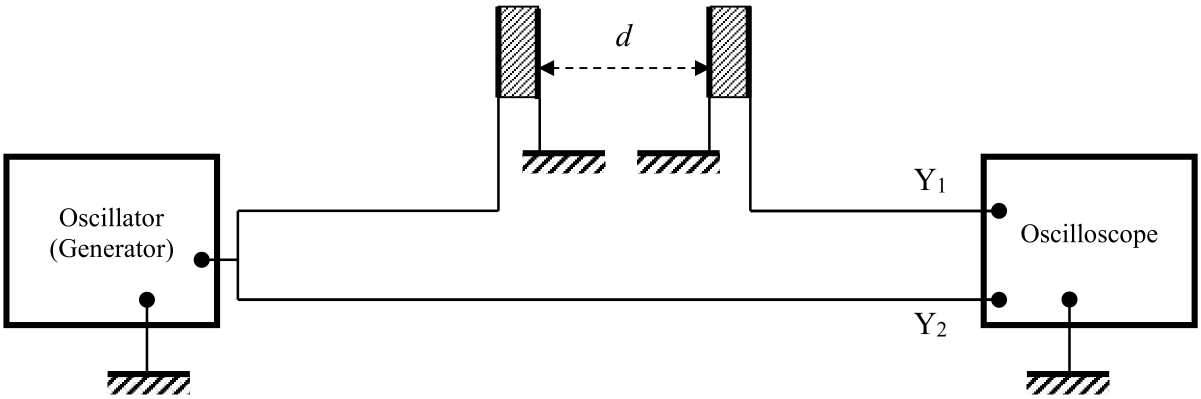

Fig. 4.3

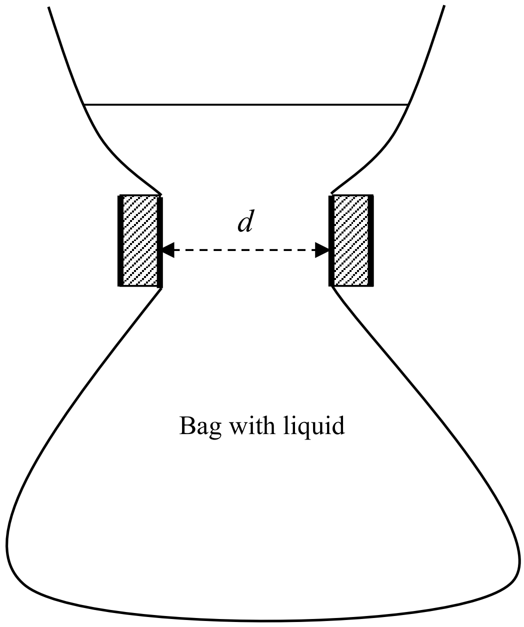

Fig. 4.4

One searches for the resonance by slowly changing the frequency of the oscillator in the range 100 – 1000 kHz and watching the signal on the oscilloscope. In this way the students could find a strong resonance at frequency $f \approx 220$ kHz. Other resonance peaks could be found at about 110 kHz and 670 kHz. They should have been neglected as they are substantially weaker. (They correspond to some other modes of vibrations.) Accuracy of these measurements was 10 kHz (due to the width of the resonance and the accuracy of the scale on the generator).

B. The ultrasonic waves pass through the liquid and generate an electric signal in the receiver. Using the same set-up (Fig. 4.3 and 4.4) we can measure dependence of the phase shift between the signals at Y_1 and Y_2 vs. distance between the piezoelectric discs d at the constant frequency found in point **A**. This phase shift is $\Delta\varphi = 2\pi df / v_l + \varphi_0$, where v_l denotes velocity of ultrasound in the liquid. φ_0 denotes a constant phase shift occurring when ultrasound passes trough the bag walls (possibly zero). The graph representing dependence

$d(\Delta\varphi)$ should be a straight line. Its slope allows to determine v_l and its error. In general, the measurements of $\Delta\varphi$ are difficult for many reflections in the bag, which perturb the signal. One of the best ways is to measure d only for $\Delta\varphi = n\pi$ (n - integer) as such points can be found rather easy. Many technical details concerning measurements can be found in [3] (pp. 37 and 38).

The liquids given to the students were water and glycerin. In the standard solution the author of the problem received the following values:

$$v_{\text{water}} = (1.50 \pm 0.10) \cdot 10^3 \text{ m/s}; \quad v_{\text{glycerin}} = (1.96 \pm 0.10) \cdot 10^3 \text{ m/s}.$$

The ratio of these values was 1.31 ± 0.15.

The ultrasonic waves are partly reflected or scattered by the walls of the bag. This effect somewhat affects measurements of the phase shift. To minimize its role one can measure the phase shift (for a given distance) or distance (at the same phase shift) several times, each time changing the shape of the bag. As regards errors in determination of velocities it is worth to mention that the most important factor affecting them was the error in determination of the frequency. This error, however, practically does not affect the ratio of velocities.

Marking Scheme

Frequency estimation

1. Formula	1 point
2. Result (with units)	1 point
3. Method of experimental determining the resonance frequency	1 point
4. Result (if within 5% of standard value)	2 points
5. Error	1 point

Measurements of velocities

1. Explanation of the method	2 points
2. Proper number of measurements in each series	3 points
3. Result for velocity in the first liquid (if within 5% of standard value)	2 points
4. Error of the above	1 point
5. Result for velocity in the second liquid (if within 5% of standard value)	2 points
6. Error of the above	1 point

Ratio of velocities

1. Result (if within 3% of standard value)	2 points
2. Error of the above	1 point

Typical mistakes

The results of this problem were very good (more than a half of competitors obtained more than 15 points). Nevertheless, many students encountered some difficulties in estimation of the frequency. Some of them assumed presence of nodes at the flat surfaces of the discs (this assumption is not adequate to the situation, but accidentally gives proper formula). In part B some students tried to find distances between nodes and antinodes for ultrasonic standing wave in the liquid. This approach gave false results as the pattern of standing waves in the bag is very complicated and changes when the shape of the bag is changed.

Acknowledgement

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Literature

[1] **Waldemar Gorzkowski** and **Andrzej Kotlicki**, *XX Międzynarodowa Olimpiada Fizyczna - cz. I*, Fizyka w Szkole *nr* **1/90**, pp. 34 - 39

[2] **Waldemar Gorzkowski**, *XX Międzynarodowa Olimpiada Fizyczna - cz. II*, Fizyka w Szkole *nr* **2/3-90**, pp. 23 - 32

[3] ***XX International Physics Olympiad*** *- Proceedings of the XX International Physics Olympiad, Warsaw (Poland), July 16 - 24, 1989*, ed. by W. Gorzkowski, World Scientific Publishing Company, Singapore 1990 [ISBN 981-02-0084-6]