THEORY 3
Recombination of Positive and Negative Ions in Ionized Gas

Introduction

A gas consists of positive ions of some element (at high temperature) and electrons. The positive ion belongs to an atom of unknown mass number Z. It is known that this ion has only one electron in the shell (orbit).

Let this ion be represented by the symbol $A^{(Z-1)+}$

Constants:

electric field constant	$\varepsilon_O = 8{,}85 \cdot 10^{-12} \frac{A \cdot s}{V \cdot m}$
elementary charge	$e = \pm 1{,}602 \cdot 10^{-19}\ A \cdot s$
	$q^2 = \frac{e^2}{4 \cdot \pi \cdot \varepsilon_O} = 2{,}037 \cdot 10^{-28}\ J \cdot m$
Planck´s constant	$\hbar = 1{,}054 \cdot 10^{-34}\ J \cdot s$
(rest) mass of an electron	$m_e = 9{,}108 \cdot 10^{-31}\ kg$
Bohr´s atomic radius	$r_B = \frac{\hbar}{m \cdot q^2} = 5{,}92 \cdot 10^{-11}\ m$
Rydberg´s energy	$E_R = \frac{q^2}{2 \cdot r_B} = 2{,}180 \cdot 10^{-18}\ J$
(rest) mass of a proton	$m_P \cdot c^2 = 1{,}503 \cdot 10^{-10}\ J$

Questions:

3.1

Assume that the ion which has just one electron left the shell.
$A^{(Z-1)+}$ is in the ground state.

In the lowest energy state, the square of the average distance of the electron from the nucleus or r^2 with components along x-, y- and z-axis being $(\Delta x)^2$, $(\Delta y)^2$ and $(\Delta z)^2$ respectively and $r_O^2 = (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2$ and also the square of the average momentum by $p_O^2 = (\Delta p_x)^2 + (\Delta p_y)^2 + (\Delta p_z)^2$, whereas $\Delta p_x \geq \frac{\hbar}{2 \cdot \Delta x}$, $\Delta p_y \geq \frac{\hbar}{2 \cdot \Delta y}$ and $\Delta p_z \geq \frac{\hbar}{2 \cdot \Delta z}$.

Write inequality involving $(p_O)^2 \cdot (r_O)^2$ in a complete form.

3.2

The ion represented by $A^{(Z-1)+}$ may capture an additional electron and consequently emits a photon.
Write down an equation which is to be used for calculation the frequency of an emitted photon.

3.3

Calculate the energy of the ion $A^{(Z-1)+}$ using the value of the lowest energy. The calculation should be approximated based on the following principles:

3.3.A

The potential energy of the ion should be expressed in terms of the average value of $\frac{1}{r}$.

(ie. $\frac{1}{r_0}$; r_0 is given in the problem).

3.3.B

In calculating the kinetic energy of the ion, use the average value of the square of the momentum given in 3.1 after being simplified by $(p_0)^2 \cdot (r_0)^2 \approx (\hbar)^2$

3.4

Calculate the energy of the ion $A^{(Z-2)+}$ taken to be in the ground state, using the same principle as the calculation of the energy of $A^{(Z-1)+}$. Given the average distance of each of the two electrons in the outermost shell (same as r_0 given in 3.3) denoted by r_1 and r_2, assume the average distance between the two electrons is given by r_1+r_2 and the average value of the square of the momentum of each electron obeys the principle of uncertainty ie.

$p_1^2 \cdot r_1^2 \approx \hbar^2$ and $p_2^2 \cdot r_2^2 \approx \hbar^2$

hint: Make use of the information that in the ground state $r_1 = r_2$

3.5

Consider in particular the ion $A^{(Z-2)+}$ is at rest in the ground state when capturing an additional electron and the captured electron is also at rest prior to the capturing. Determine the numerical value of Z, if the frequency of the emitted photon accompanying electron capturing is $2{,}057 \, . \, 10^{17}$ rad/s. Identify the element which gives rise to the ion.