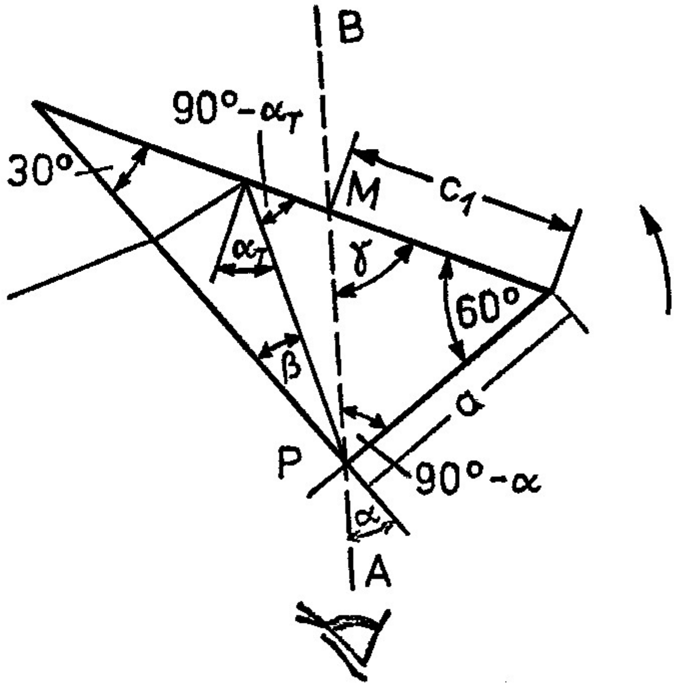
Solution of problem 4:

a) Calculation of the refractive index of the prism

First method:



Draw a straight line A – B on a sheet of paper and let this be your line of sight. Place the prism with its rectangular edge facing you onto the line (at point P on the line). Now turn the prism in the direction of the arrow until the dark edge of total reflection which can be seen in the short face of the prism coincides with the 90° edge of the prism. Mark a point M and measure the length c_1. Measure also the length of the short face of the prism.

The following equations apply:

$$\sin\alpha_T = \frac{1}{n_p} \tag{1}$$

$$\frac{\sin\alpha}{\sin\beta} = n_p \tag{2}$$

$$\beta = 60° - \alpha_T \tag{3}$$

$$\gamma = 30° + \alpha \tag{4}$$

$$\frac{\sin\gamma}{\sin(90°-\alpha)} = \frac{a}{c_1} \tag{5}$$

From eq. (5) follows with eq. (4) and the given formulae:

$$\frac{a}{c_1}\cdot\cos\alpha = \sin(30°+\alpha) = \frac{1}{2}\cdot\cos\alpha + \frac{1}{2}\cdot\sqrt{3}\cdot\sin\alpha$$

$$\sin\alpha = \frac{2a - c_1}{2\cdot\sqrt{a^2 - a\cdot c_1 + c_1^{\,2}}} \tag{6}$$

From eqs. (2), (3) and (1) follows:

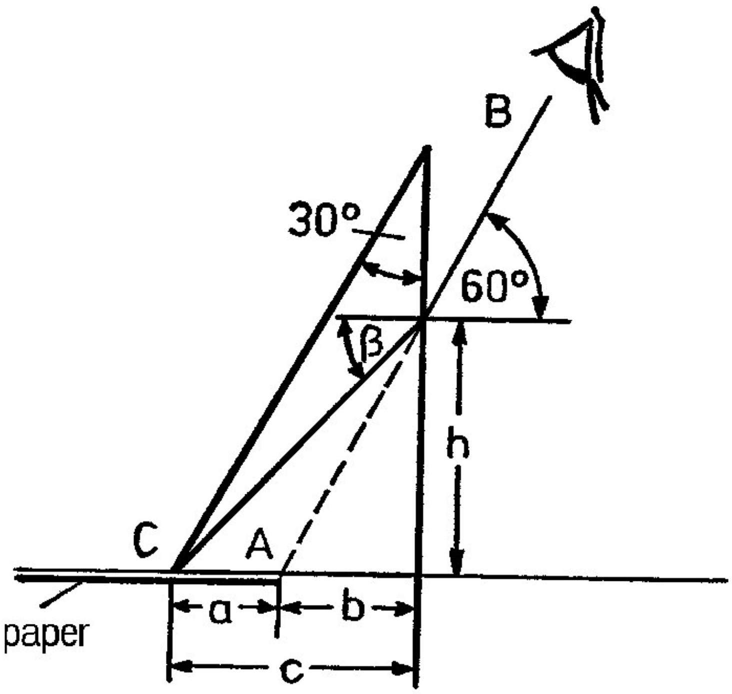
$$\sin\alpha = n_p\cdot\sin(60°-\alpha_T) = \frac{n_p}{2}\cdot\left(\sqrt{3}\cdot\cos\alpha_T - \sin\alpha_T\right)$$

$$n_p = +\left\{\frac{1}{3}\cdot(2\cdot\sin\alpha+1)^2+1\right\}^{1/2} \tag{7}$$

When measuring c_1 and a one notices that within the error limits of ± 1 mm a equals c_1.

Hence: $\sin\alpha = \frac{1}{2}$ and $n_p = 1.53$. (8)

Second method:



Place edge C of the prism on edge A of a sheet of paper and look along the prism hypotenuse at edge A so that your direction of sight B-A and the table surface form an angle of 60°. Then shift the prism over the edge of the paper into the position shown, such that prism edge C can be seen inside the prism collinear with edge A of the paper outside the prism. The direction of sight must not be changed while the prism is being displaced.

The following equations apply:

$$\left.\begin{array}{l}\tan\beta = \frac{h}{c} \\ \tan 60° = \sqrt{3} = \frac{h}{b}\end{array}\right\} \Rightarrow h = b\cdot\sqrt{3} = \frac{c\cdot\sin\beta}{\sqrt{1-\sin^2\beta}} \tag{9}$$

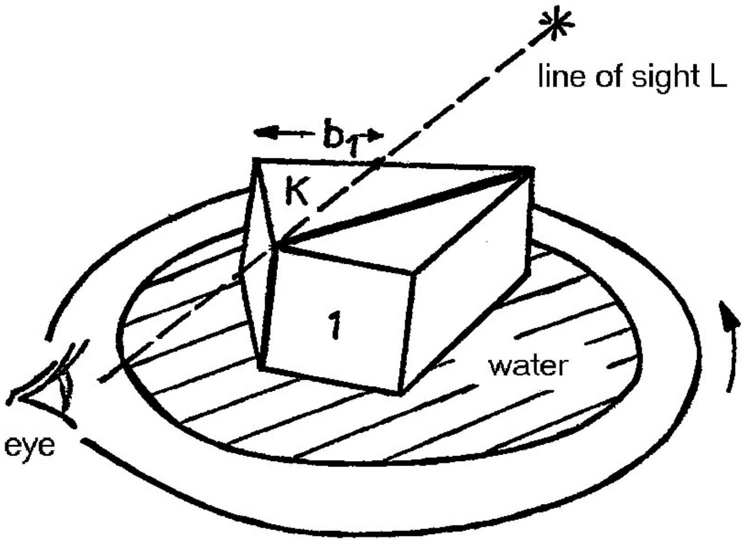
$$\sin\beta = \sin 60°\cdot\frac{1}{n_p} = \frac{\sqrt{3}}{2\cdot n_p} \tag{10}$$

$$n_p = \frac{1}{2}\cdot\sqrt{\left(\frac{c}{b}\right)^2 + 3} \tag{11}$$

With the measured values c = 29 mm and b = 11.5 mm, it follows

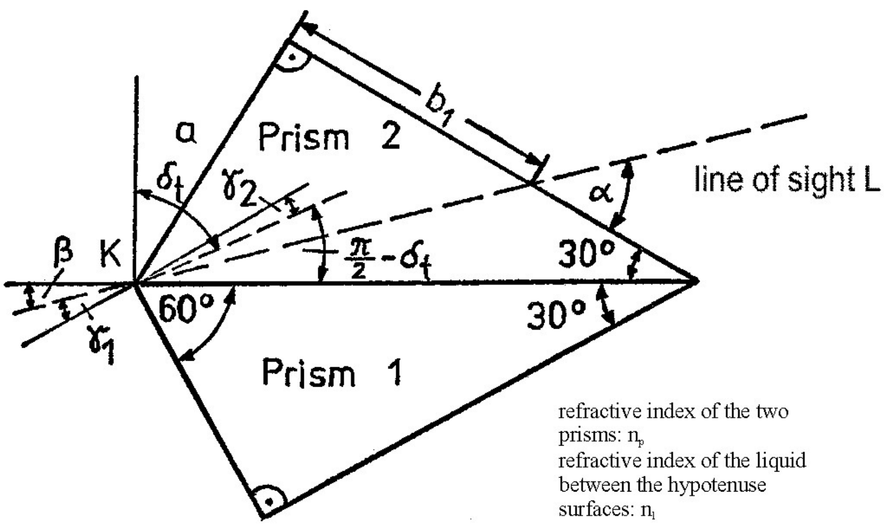
$$n_p = 1.53. \tag{12}$$

b) Determination of the refractive index of the liquid by means of two prisms



Place the two prisms into a glass dish filled with water as shown in the figure above. Some water will rise between the hypotenuse surfaces. By pressing and moving the prisms slightly against each other the water can be made to cover the whole surface. Look over the 60° edges of the prisms along a line of sight L (e.g. in the direction of a fixed point on an illuminated wall). Turn the glass dish together with the two prisms in such a way that the dark shadow of total reflection which can be seen in the short face of prism 1 coincides with the 60° edge of that prism (position shown in the figure below).

While turning the arrangement take care to keep the 60° edge (point K) on the line of sight L. In that position measure the length b_1 with a ruler (marking, reading). The figure below illustrates the position described.



refractive index of the two prisms: n_p
refractive index of the liquid between the hypotenuse surfaces: n_l

If the refractive index of the prism is known (see part a) the refractive index of the liquid may be calculated as follows:

$$\sin\alpha = \frac{a}{\sqrt{a^2 + b_1^{\,2}}} \tag{13}$$

$$\beta = \alpha - 30°;\ \ \gamma_1 = 30° - \beta = 60° - \alpha \tag{14, 15}$$

$$\frac{\sin\gamma_1}{\sin\gamma_2} = n_p \ \ \text{refraction at the short face of prism 1.} \tag{16}$$

The angle of total reflection δ_t at the hypotenuse surface of prism 1 in the position described is:

$$\frac{\pi}{2} - \delta_t = 30° - \gamma_2 \tag{17}$$

$$\delta_t = 60° + \arcsin\left(\frac{\sin\gamma_1}{n_p}\right) \tag{18}$$

From this we can easily obtain n_1:

$$n_1 = n_p \cdot \sin\delta_t = n_p \cdot \sin\left\{60° + \arcsin\frac{\sin\gamma_1}{n_p}\right\} \tag{19}$$

Numerical example for water as liquid:

b_1 = 1.9 cm; α = 55.84°; γ_1 = 4.16°; δ_t = 62.77°; a = 2.8 cm; with n_p = 1.5 follows

n_1= 1.33. (20)

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