Solution of problem 2:

1. Determination of B:

   The vector of the velocity of any electron is divided into components parallel with and perpendicular to the magnetic field $\vec{B}$:

   $$\vec{v} = \vec{v}_{\parallel} + \vec{v}_{\perp} \tag{1}$$

   The Lorentz force $\vec{F} = -e \cdot (\vec{v} \times \vec{B})$ influences only the perpendicular component, it acts as a radial force:

   $$m \cdot \frac{v_{\perp}^{2}}{r} = e \cdot v_{\perp} \cdot B \tag{2}$$

   Hence the radius of the circular path that has been travelled is

   $$r = \frac{m}{e} \cdot \frac{v_{\perp}}{B} \tag{3}$$

and the period of rotation which is independent of $v_\perp$ is

$$T = \frac{2 \cdot \pi \cdot r}{v_\perp} = \frac{2 \cdot \pi \cdot m}{B \cdot e} \tag{4}$$

The parallel component of the velocity does not vary. Because of $\alpha_0 << 1$ it is approximately equal for all electrons:

$$v_{\parallel 0} = v_0 \cdot \cos\alpha_0 \approx v_0 \tag{5}$$

Hence the distance b between the focusing points, using eq. (5), is

$$b = v_{\parallel 0} \cdot T \approx v_0 \cdot T \tag{6}$$

From the law of conservation of energy follows the relation between the acceleration voltage V_0 and the velocity v_0:

$$\frac{m}{2} \cdot {v_0}^2 = e \cdot V_0 \tag{7}$$

Using eq. (7) and eq. (4) one obtains from eq. (6)

$$b = \frac{2 \cdot \pi}{B} \cdot \sqrt{2 \cdot \frac{m}{e} \cdot V_0} \tag{8}$$

and because of $b = \frac{2 \cdot \pi \cdot R}{4}$ one obtains

$$B = \frac{4}{R} \cdot \sqrt{2 \cdot \frac{m}{e} \cdot V_0} = 1.48 \cdot 10^{-2} \frac{Vs}{m^2} \tag{9}$$

2. Determination of B_1:

Analogous to eq. (2)

$$m \cdot \frac{v_0^2}{R} = e \cdot v_0 \cdot B_1 \tag{10}$$

must hold.

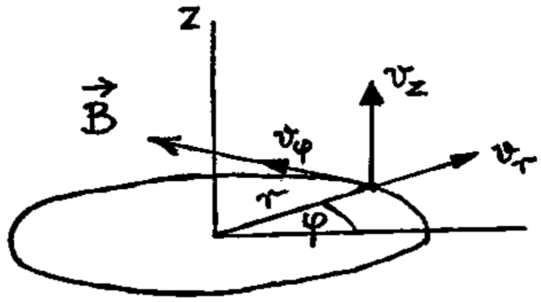
From eq. (7) follows

$$B_1 = \frac{1}{R} \cdot \sqrt{2 \cdot \frac{m}{e} \cdot V_0} = 0.37 \cdot 10^{-2} \frac{Vs}{m^2} \tag{11}$$

3. Finiteness of r_1 and direction of the drift velocity

In the magnetic field the lines of force are circles with their centres on the symmetry axis (z-axis) of the toroid.

In accordance with the symmetry of the problem, polar coordinates r and φ are introduced into the plane perpendicular to the z-axis (see figure below) and the occurring vector quantities (velocity, magnetic field $\vec{B}$, Lorentz force) are divided into the corresponding components.



Since the angle of aperture of the beam can be neglected examine a single electron injected tangentially into the toroid with velocity v_0 on radius R.

In a static magnetic field the kinetic energy is conserved, thus

$$E = \frac{m}{2}\left(v_r^2 + v_\varphi^2 + v_z^2\right) = \frac{m}{2} v_0^2 \qquad (12)$$

The radial points of inversion of the electron are defined by the condition

$$v_r = 0$$

Using eq. (12) one obtains

$$v_0^2 = v_\varphi^2 + v_z^2 \qquad (13)$$

Such an inversion point is obviously given by

$$r = R \cdot \left(v_\varphi = v_0,\ v_r = 0,\ v_z = 0\right).$$

To find further inversion points and thus the maximum radial deviation of the electron the components of velocity v_φ and v_z in eq. (13) have to be expressed by the radius.

v_φ will be determined by the law of conservation of angular momentum. The Lorentz force obviously has no component in the φ - direction (parallel to the magnetic field). Therefore it cannot produce a torque around the z-axis. From this follows that the z-component of the angular momentum is a constant, i.e. $L_z = m \cdot v_\varphi \cdot r = m \cdot v_0 \cdot R$ and

therefore $v_\varphi = v_0 \cdot \frac{R}{r}$ (14)

v_z will be determined from the equation of motion in the z-direction. The z-component of the Lorentz force is $F_z = - e \cdot B \cdot v_r$. Thus the acceleration in the z-direction is

$$a_z = -\frac{e}{m} \cdot B \cdot v_r \,. \tag{15}$$

That means, since B is assumed to be constant, a change of v_z is related to a change of r as follows:

$$\Delta v_z = -\frac{e}{m} \cdot B \cdot \Delta r$$

Because of $\Delta r = r - R$ and $\Delta v_z = v_z$ one finds

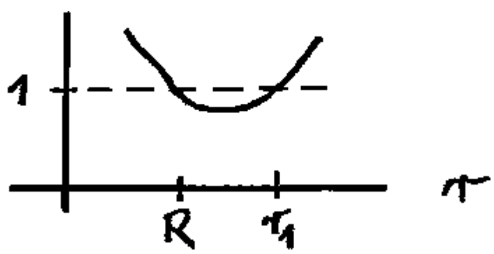
$$v_z = -\frac{e}{m} \cdot B \cdot (r - R) \tag{16}$$

Using eq. (14) and eq. (15) one obtains for eq. (13)

$$1 = \left(\frac{R}{r}\right)^2 + A^2 \cdot \left(\frac{r}{R} - 1\right)^2 \tag{17}$$

where $A = \frac{e}{m} \cdot B \cdot \frac{R}{v_0}$

Discussion of the curve of the right side of eq. (17) gives the qualitative result shown in the following diagram:



Hence r_1 is finite. Since $R \le r \le r_1$ eq. (16) yields $v_z < 0$. Hence the drift is in the direction of the negative z-axis.