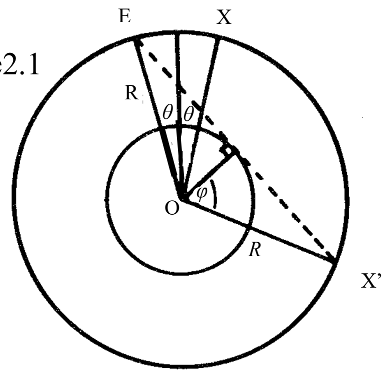
Answers Q2

2.(i)

Figure2.1



$$\text{EX} = 2R\sin\theta \quad \therefore t = \frac{2R\sin\theta}{v}$$

where $v = v_P$ for P waves and $v = v_S$ for S waves.

This is valid providing X is at an angular separation less than or equal to X', the tangential ray to the liquid core. X' has an angular separation given by, from the diagram,

$$2\phi = 2\cos^{-1}\left(\frac{R_C}{R}\right),$$

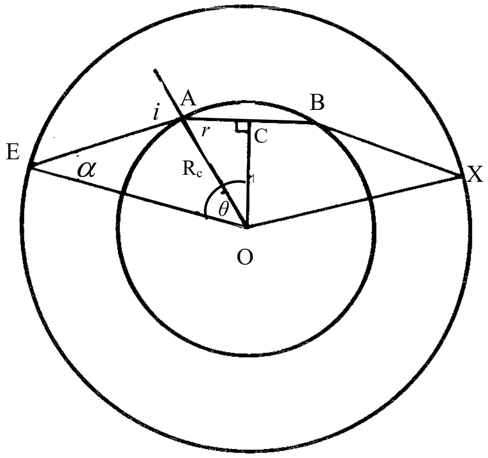
Thus

$$t = \frac{2R\sin\theta}{v} \quad , \qquad \text{for } \theta \le \cos^{-1}\left(\frac{R_C}{R}\right),$$

where $v = v_P$ for P waves and $v = v_S$ for shear waves.

(ii) $\dfrac{R_C}{R} = 0.5447$ and $\dfrac{v_{CP}}{v_P} = 0.831.3$

Figure 2.2



From Figure 2.2

$$\theta = A\hat{O}C + E\hat{O}A \Rightarrow \theta = (90 - r) + (1 - \alpha) \qquad (1)$$

(ii) Continued

Snell's Law gives:

$$\frac{\sin i}{\sin r} = \frac{v_P}{v_{CP}}. \qquad (2)$$

From the triangle EAO, sine rule gives

$$\frac{R_C}{\sin x} = \frac{R}{\sin i}. \qquad (3)$$

Substituting (2) and (3) into (1)

$$\theta = \left[90 - \sin^{-1}\left(\frac{v_{CP}}{v_P} \sin i \right) + i - \sin^{-1}\left(\frac{R_C}{R} \sin i \right) \right] \qquad (4)$$

(iii)

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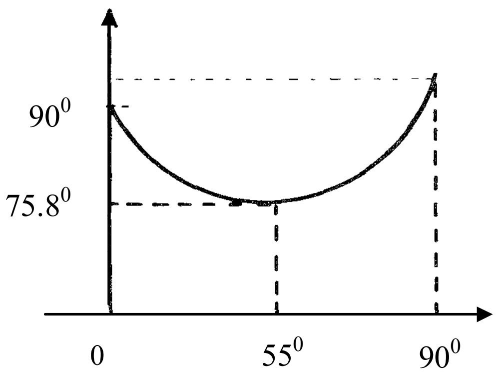
For minimum $\theta, \dfrac{d\theta}{di} = 0$. $\Rightarrow 1 - \dfrac{\left(\dfrac{v_{CP}}{v_P}\right)\cos i}{\sqrt{1 - \left(\dfrac{v_{CP}}{v_P}\sin i\right)^2}} - \dfrac{\left(\dfrac{R_C}{R}\right)\cos i}{\sqrt{1 - \left(\dfrac{R_C}{R}\sin i\right)^2}} = 0$

Substituting $i = 55.0^\circ$ gives LHS=0, this verifying the minimum occurs at this value of i. Substituting $i = 55.0^\circ$ into (4) gives $\theta = 75.8^0$.

Plot of θ against i.



Substituting into 4:

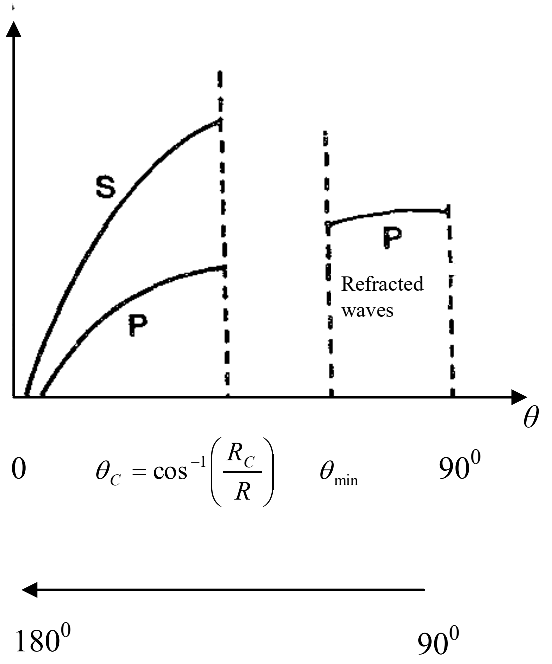
$i = 0$ gives $\theta = 90$

$i = 90°$ gives $\theta = 90.8°$

Substituting numerical values for $i = 0 \rightarrow 90°$ one finds a minimum value at $i = 55^\circ$; the minimum values of 0, $\theta_{MIN} = 75•8°$.

Physical Consequence

As θ has a minimum value of 75•8° observers at position for which 2 θ <151•6° will not observe the earthquake as seismic waves are not deviated by angles of less than 151•6°. However for 2 $\theta \le$ 114° the direct, non-refracted, seismic waves will reach the observer.



(iv) Using the result

$$t = \frac{2r\sin\theta}{v}$$

the time delay Δt is given by

$$\Delta t = 2R\sin\theta\left[\frac{1}{v_S} - \frac{1}{v_P}\right]$$

Substituting the given data

$$131 = 2(6370)\left[\frac{1}{6.31} - \frac{1}{10.85}\right]\sin\theta$$

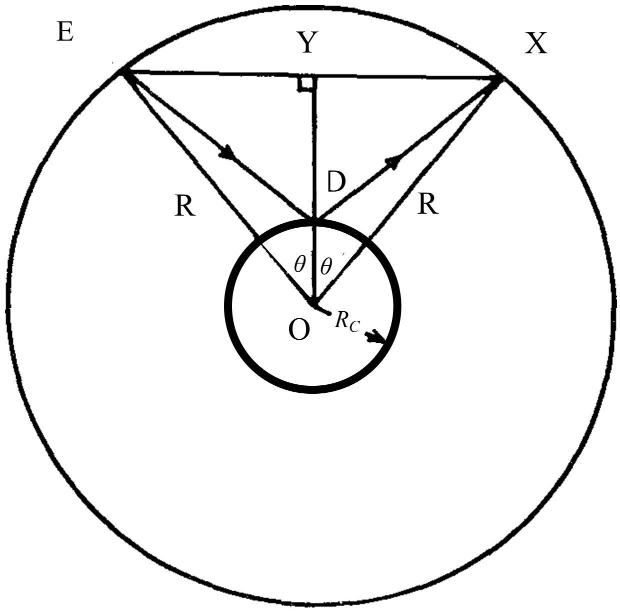
Therefore the angular separation of E and X is

$$2\theta = 17.84^o$$

This result is less than $2\cos^{-1}\left(\frac{R_C}{R}\right) = 2\cos^{-1}\left(\frac{3470}{6370}\right) = 114^o$

And consequently the seismic wave is not refracted through the core.

(v)



The observations are most likely due to reflections from the mantle-core interface. Using the symbols given in the diagram, the time delay is given by

$$\Delta t' = (\mathrm{ED} + \mathrm{DX})\left[\frac{1}{v_S} - \frac{1}{v_P}\right]$$

$$\Delta t' = 2(\mathrm{ED})\left[\frac{1}{v_S} - \frac{1}{v_P}\right] \text{ as ED = EX by symmetry}$$

In the triangle EYD,

$$(\mathrm{ED})^2 = (R\sin\theta)^2 + (R\cos\theta - R_C)^2$$

$$(\mathrm{ED})^2 = R^2 + R_C^2 - 2RR_C\cos\theta \qquad \sin^2\theta + \cos^2\theta = 1$$

Therefore

$$\Delta t' = 2\sqrt{R^2 + R_C^2 - 2RR_C\cos\theta}\left[\frac{1}{v_S} - \frac{1}{v_P}\right]$$

Using (ii)

$$\Delta t' = \frac{\Delta t}{R\sin\theta}\sqrt{R^2 + R_C{}^2 - 2RR_C\cos\theta}$$

$$\Rightarrow 396.7s \text{ or 6m 37s}$$

Thus the subsequent time interval, produced by the reflection of seismic waves at the mantle core interface, is consistent with angular separation of 17.84^0.