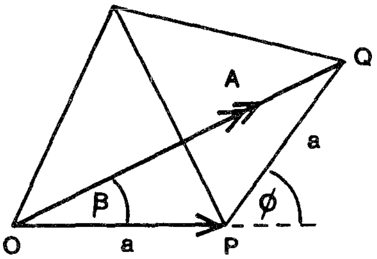
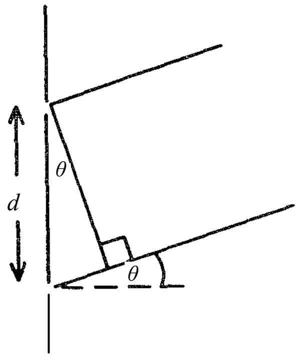
Answers Question 1

(i) Vector Diagram





If the phase of the light from the first slit is zero, the phase from second slit is

$$\phi = \frac{2\pi}{\lambda} d \sin\theta$$

Adding the two waves with phase difference ϕ where $\xi = 2\pi\left(ft - \frac{x}{\lambda}\right)$,

$$a\cos(\xi+\phi) + a\cos(\xi) = 2a\cos(\phi/2)(\xi+\phi/2)$$
$$a\cos(\xi+\phi) + a\cos(\xi) = 2a\cos\beta\{\cos(\xi+\beta)\}$$

This is a wave of amplitude $A = 2a\cos\beta$ and phase β. From vector diagram, in isosceles triangle OPQ,

$$\beta = \frac{1}{2}\phi = \frac{\pi}{\lambda} d\sin\theta \qquad (NB \quad \phi = 2\beta)$$

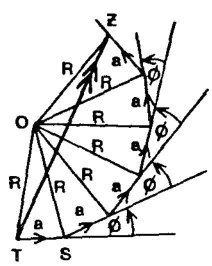
and

$$A = 2a\cos\beta.$$

Thus the sum of the two waves can be obtained by the addition of two vectors of amplitude a and angular directions 0 and ϕ.

(ii) Each slit in diffraction grating produces a wave of amplitude a with phase 2β relative to previous slit wave. The vector diagram consists of a 'regular' polygon with sides of constant length a and with constant angles between adjacent sides.
Let O be the centre of circumscribing circle passing through the vertices of the polygon. Then radial lines such as OS have length R and bisect the internal angles of the polygon. Figure 1.2.

Figure 1.2



$$O\hat{S}T = O\hat{T}S = \frac{1}{2}(180-\phi)$$

$$\text{and} \quad T\hat{O}S = \phi$$

In the triangle *TOS*, for example

$$a = 2R\sin(\phi/2) = 2R\sin\beta \ \text{ as } \ (\phi = 2\beta)$$

$$\therefore R = \frac{a}{2\sin\beta} \qquad (1)$$

As the polygon has N faces then:

$$T\hat{O}Z = N(T\hat{O}Z) = N\phi = 2N\beta$$

Therefore in isosceles triangle TOZ, the amplitude of the resultant wave, TZ, is given by

$$2R\sin N\beta\,.$$

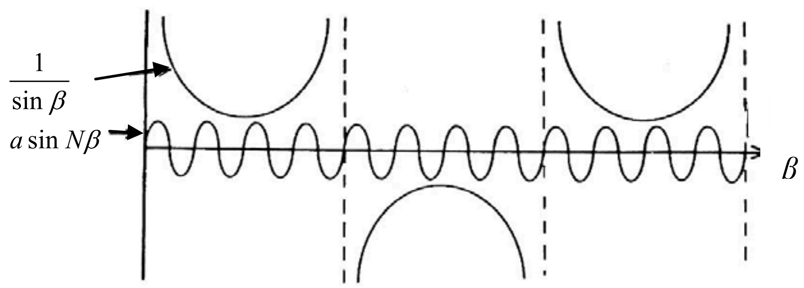
Hence form (1) this amplitude is

$$\frac{a\sin N\beta}{\sin\beta}$$

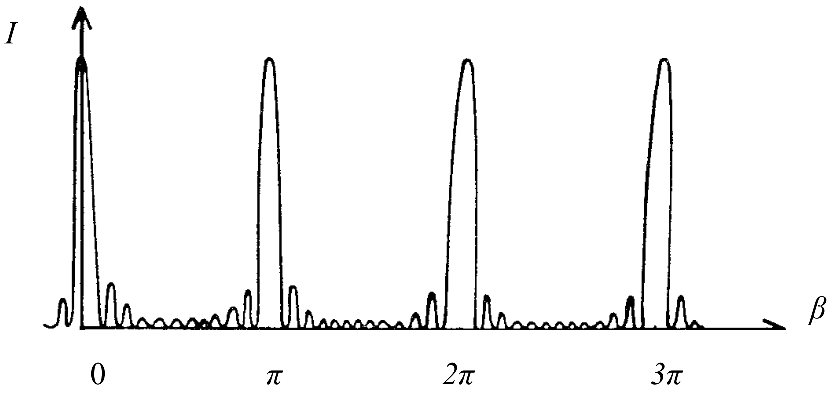
Resultant phase is

$$\begin{aligned}
&= Z\hat{T}S \\
&= O\hat{T}S - O\hat{T}Z \\
&\left(90-\frac{\phi}{2}\right)-\frac{1}{2}(180-N\phi) \\
&-\frac{1}{2}(N-1)\phi \\
&= (N-1)\beta
\end{aligned}$$

(iii)



Intensity $I = \dfrac{a^2\sin^2 N\beta}{\sin^2\beta}$



(iv) For the principle maxima $\beta = \pi p$ where $p = 0 \pm 1 \pm 2$........

$$I_{\max} = a^2\left(\frac{N\beta'}{\beta'}\right) = N^2a^2 \qquad \beta' = 0 \text{ and } \beta = \pi p + \beta'$$

(v) Adjacent max. estimate I_1 :

$$\sin^2 N\beta = 1, \ \ \beta = 2\pi p \mp \frac{3\pi}{2N} \text{ i.e } \beta = \pm\frac{3\pi}{2N}$$

$\left[\beta = \pi p \pm \frac{\pi}{2N}\right]$ does not give a maximum as can be observed from the graph.

$$I_1 = a^2 \frac{1}{\frac{3\pi^2}{2n}} = \frac{a^2N^2}{23} \text{ for } N \gg 1$$

Adjacent zero intensity occurs for $\beta = \pi\rho \pm \frac{\pi}{N}$ i.e. $\delta = \pm\frac{\pi}{N}$

For phase differences much greater than δ, $\mathrm{I} = \mathrm{a}^2\left(\frac{\sin N\beta}{\sin\beta}\right) = a^2$.

(vi)

$\beta = n\pi$ for a principle maximum

i.e. $\frac{\pi}{\lambda} d\sin\theta = n\pi \qquad n = 0, \pm1, \pm2$..........

Differentiating w.r.t, λ

$d\cos\theta\Delta\theta = n\Delta\lambda$

$$\Delta\theta = \frac{n\Delta\lambda}{d\cos\theta}$$

Substituting $\lambda = 589.0\text{nm}$, $\lambda + \Delta\lambda = 589.6\,\text{nm}$. n = 2 and $d = 1.2\times10^{-6}\,\text{m}$.

$$\Delta\theta = \frac{n\Delta\lambda}{d\sqrt{1-\left(\frac{n\lambda}{d}\right)^2}} \quad \text{as } \sin\theta = \frac{n\lambda}{d} \text{ and } \cos\theta = \sqrt{1-\left(\frac{n\lambda}{d}\right)^2}$$

$\Rightarrow \Delta\theta = 5.2\times10^{-3}\,rads$ or 0.30^0