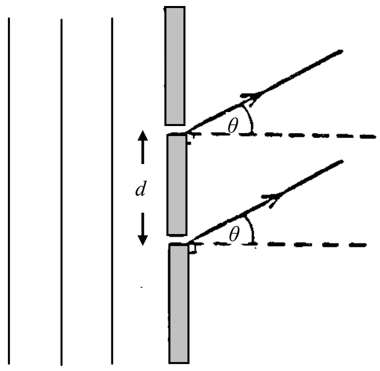
Q1

Figure 1.1



A plane monochromatic light wave, wavelength λ and frequency *f*, is incident normally on two identical narrow slits, separated by a distance *d*, as indicated in Figure 1.1. The light wave emerging at each slit is given, at a distance *x* in a direction θ at time *t*, by

$$y = a\cos[2\pi(ft - x/\lambda)]$$

where the amplitude *a* is the same for both waves. (Assume *x* is much larger than *d*).

(i) Show that the two waves observed at an angle θ to a normal to the slits, have a resultant amplitude A which can be obtained by adding two vectors, each having magnitude a, and each with an associated direction determined by the phase of the light wave.

Verify geometrically, from the vector diagram, that

$$A = 2a\cos\theta$$

where

$$\beta = \frac{\pi}{\lambda} d\sin\theta$$

(ii) The double slit is replaced by a diffraction grating with *N* equally spaced slits, adjacent slits being separated by a distance *d*. Use the vector method of adding amplitudes to show that the vector amplitudes, each of magnitude *a*, form a part of a regular polygon with vertices on a circle of radius *R* given by

$$R = \frac{a}{2\sin\beta},$$

Deduce that the resultant amplitude is

$$\frac{a\sin N\beta}{\sin\beta}$$

and obtain the resultant phase difference relative to that of the light from the slit at the edge of the grating.

(iii) Sketch, in the same graph, sin $N\beta$ and $(1/\sin\beta)$ as a function of β. On a separate graph show how the intensity of the resultant wave varies as a function of β.

(iv) Determine the intensities of the principal intensity maxima.

(v) Show that the number of principal maxima cannot exceed

$$\left(\frac{2d}{\lambda}+1\right)$$

(vi) Show that two wavelengths λ and $\lambda+\delta\lambda$, where $\delta\lambda << \lambda$, produce principal maxima with an angular separation given by

$$\Delta\theta = \frac{n\Delta\lambda}{d\cos\theta} \qquad \text{where } n = 0,\ \pm 1,\ \pm 2\text{....etc}$$

Calculate this angular separation for the sodium D lines for which

$$\lambda = 589.0\text{nm}, \quad \lambda+\Delta\lambda = 589.6\text{nm}, \quad n = 2, \text{ and } d = 1.2\times 10^{-6}\text{ m.}$$

$$\left[\text{reminder:} \quad \cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right).\cos\left(\frac{\text{A - B}}{2}\right)\right]$$