2 Solutions

2.1 Theoretical competition

Problem 1

a) Let the electrical signals supplied to rods 1 and 2 be $E_1 = E_0 \cos \omega t$ and $E_2 = E_0 \cos(\omega t + \delta)$, respectively. The condition for a maximum signal in direction ϑ_A (Fig. 4) is:

$$\frac{2\pi a}{\lambda} \sin \vartheta_A - \delta = 2\pi N$$

and the condition for a minimum signal in direction ϑ_B:

$$\frac{2\pi a}{\lambda} \sin \vartheta_B - \delta = 2\pi N' + \pi \qquad \text{(2p.)}$$

where N and N' are arbitrary integers. In addition, $\vartheta_A - \vartheta_B = \varphi$, where

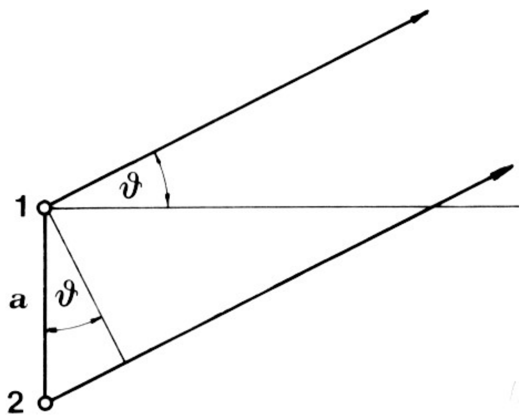

Fig. 4

φ is given. The problem can now be formulated as follows: Find the parameters a, ϑ_A, ϑ_B, δ, N, and N' satisfying the above equations such, that a is minimum.

We first eliminate δ by subtracting the second equation from the first one:

$$a \sin \vartheta_A - a \sin \vartheta_B = \lambda(N - N' - \tfrac{1}{2}) .$$

Using the sine addition theorem and the relation $\vartheta_B = \vartheta_A - \varphi$:

$$2a\cos(\vartheta_A - \tfrac{1}{2}\varphi)\sin\tfrac{1}{2}\varphi = \lambda(N - N' - \tfrac{1}{2})$$

or

$$a = \frac{\lambda(N - N' - \frac{1}{2})}{2\cos(\vartheta_A - \frac{1}{2}\varphi)\sin\frac{1}{2}\varphi}\,.$$

The minimum of a is obtained for the greatest possible value of the denominator, i. e.:

$$\cos(\vartheta_A - \tfrac{1}{2}\varphi) = 1\,, \qquad \vartheta_A = \tfrac{1}{2}\varphi\,,$$

and the minimum value of the numerator, i. e.:

$$N - N' = 1\,.$$

The solution is therefore:

$$a = \frac{\lambda}{4\sin\frac{1}{2}\varphi}\,, \quad \vartheta_A = \tfrac{1}{2}\varphi\,, \quad \vartheta_B = -\tfrac{1}{2}\varphi \quad \text{and} \quad \delta = \tfrac{1}{2}\pi - 2\pi N\,. \qquad \text{(6p.)}$$

(N = 0 can be assumed throughout without loosing any physically relevant solution.)

b) The wavelength $\lambda = c/\nu = 11.1$ m, and the angle between directions A and B, $\varphi = 157^\circ - 72^\circ = 85^\circ$. The minimum distance between the rods is $a = 4.1$ m, while the direction of the symmetry line of the rods is $72^\circ + 42.5^\circ = 114.5^\circ$ measured from the north. (2 p.)