

Solution:

In the coordinate system of the figure, we have for the centre of mass coordinates of the two triangular parts of the water

$$(x_1, y_1) = (L/3, h/2 + \xi/3) \quad (x_2, y_2) = (-L/3, h/2 - \xi/3).$$

For the entire water mass the centre of mass coordinates will then be

$$(x_{CoM}, y_{CoM}) = \left(\frac{\xi L}{6h}, \frac{\xi^2}{6h} \right)$$

Due to that the y component is quadratic in ξ will be much much smaller than the x component.

The velocities of the water mass are

$$(v_x, v_y) = \left(\frac{\partial x}{\partial \xi}, \frac{\partial y}{\partial \xi} \right),$$

and again the vertical component is much smaller than the horizontal one.

We now in our model neglect the vertical components. The total energy (kinetic + potential) will then be

$$W = W_k + W_p = \frac{1}{2} M \frac{\xi^2 L^2}{36h^2} + Mg \frac{\xi^2}{6h}$$

For a harmonic oscillator we have

$$W = W_k + W_p = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} m \omega^2 x^2$$

Identifying gives

$$\omega = \sqrt{\frac{12gh}{L}} \quad \text{or} \quad T_{model} = \frac{\pi L}{\sqrt{3h}}.$$

Comparing with the experimental data we find $T_{experiment} \approx 1.1 \cdot T_{model}$, our model gives a slight underestimation of the oscillation period.

Applying our corrected model on the Vättern data we have that the oscillation period of the seiche is about 3 hours.

Many other models are possible and give equivalent results.

