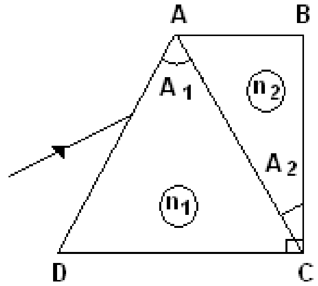
Optics – Problem III (7points)

Prisms

Two dispersive prisms having apex angles $\hat{A}_1 = 60°$ and $\hat{A}_2 = 30°$ are glued as in the figure ($\hat{C} = 90°$). The dependences of refraction indexes of the prisms on the wavelength are given by the relations

$$n_1(\lambda) = a_1 + \frac{b_1}{\lambda^2};$$

$$n_2(\lambda) = a_2 + \frac{b_2}{\lambda^2}$$



were

$a_1 = 1{,}1$, $b_1 = 1\cdot 10^5\, nm^2$, $a_2 = 1{,}3$, $b_2 = 5\cdot 10^4\, nm^2$.

- a. Determine the wavelength λ_0 of the incident radiation that pass through the prisms without refraction on AC face at any incident angle; determine the corresponding refraction indexes of the prisms.
- *b.* Draw the ray path in the system of prisms for three different radiations λ_{red}, λ_0, λ_{violet} incident on the system at the same angle.
- *c.* Determine the minimum deviation angle in the system for a ray having the wavelength λ_0.
- *d.* Calculate the wavelength of the ray that penetrates and exits the system along directions parallel to DC.

Problem III - Solution

a. The ray with the wavelength λ_0 pass trough the prisms system without refraction on AC face at any angle of incidence if :

$$n_1(\lambda_0) = n_2(\lambda_0)$$

Because the dependence of refraction indexes of prisms on wavelength has the form :

$$n_1(\lambda) = a_1 + \frac{b_1}{\lambda^2} \quad (3.1)$$

$$n_2(\lambda) = a_2 + \frac{b_2}{\lambda^2} \quad (3.2)$$

The relation (3.1) becomes:

$$a_1 + \frac{b_1}{\lambda_0^2} = a_2 + \frac{b_2}{\lambda_0^2} \quad (3.3)$$

The wavelength λ_0 has correspondingly the form:

$$\lambda_0 = \sqrt{\frac{b_1 - b_2}{a_2 - a_1}} \quad (3.4)$$

Substituting the furnished numerical values

$$\lambda_0 = 500\, nm \quad (3.5)$$

The corresponding common value of indexes of refraction of prisms for the radiation with the wavelength λ_0 is:

$$n_1(\lambda_0) = n_2(\lambda_0) = 1,5 \qquad (3.6)$$

The relations (3.6) and (3.7) represent the answers of question **a.**

b. For the rays with different wavelength (λ_{red} , λ_0 , λ_{violet}) having the same incidence angle on first prism, the paths are illustrated in the figure 1.1.

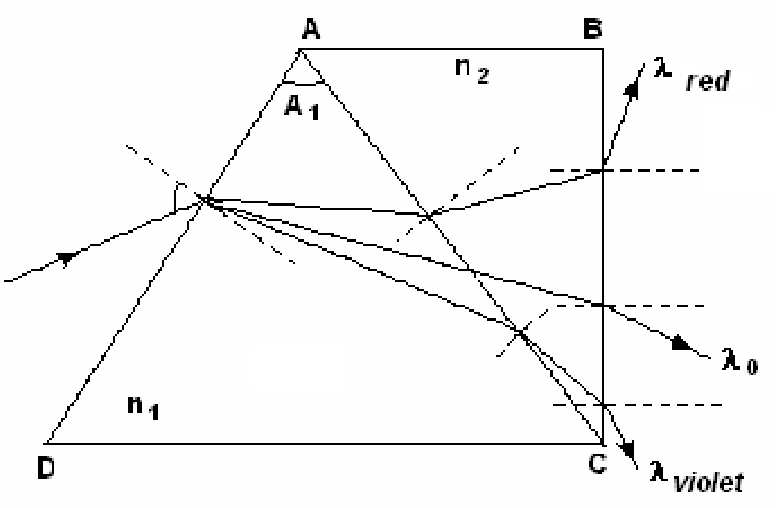


Figure 3.1

The draw illustrated in the figure 1.1 represents the answer of question **b.**

c. In the figure 1.2 is presented the path of ray with wavelength λ_0 at minimum deviation (the angle between the direction of incidence of ray and the direction of emerging ray is minimal).

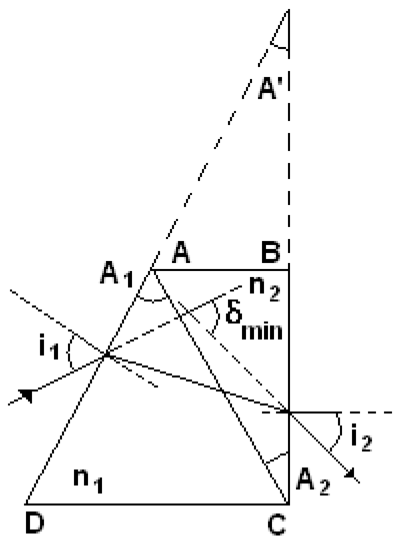


Figure 3.2

In this situation

$$n_1(\lambda_0) = n_2(\lambda_0) = \frac{\sin\frac{\delta_{\min} + A'}{2}}{\sin\frac{A'}{2}} \qquad (3.7)$$

where

$m(\hat{A}') = 30°$,

as in the figure 1.1

Substituting in (3.8) the values of refraction indexes the result is

$$\sin\frac{\delta_{\min} + A'}{2} = \frac{3}{2}\cdot\sin\frac{A'}{2} \qquad (3.8)$$

or

$$\delta_{\min} = 2\arcsin\left(\frac{3}{2}\cdot\sin\frac{A'}{2}\right) - \frac{A'}{2} \qquad (3.9)$$

Numerically

$$\delta_{\min} \cong 30{,}7° \qquad (3.10)$$

The relation (3.11) represents the answer of question **c**.

d. Using the figure 1.3 the refraction law on the AD face is

$$\sin i_1 = n_1 \cdot \sin r_1 \qquad (3.11)$$

The refraction law on the AC face is

$$n_1 \cdot \sin r_1' = n_2 \cdot \sin r_2 \qquad (3.12)$$

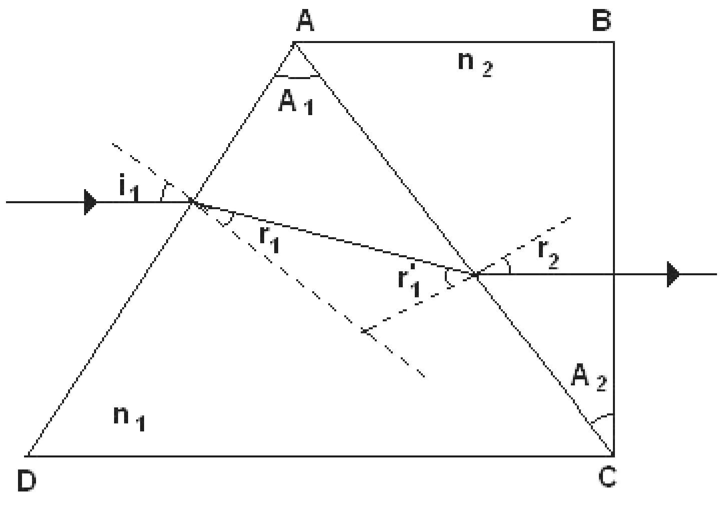


Figure 3.3

As it can be seen in the figure 1.3

$$r_2 = A_2 \qquad (3.13)$$

and

$$i_1 = 30° \qquad (3.14)$$

Also,

$$r_1 + r_1' = A_1 \qquad (3.15)$$

Substituting (3.16) and (3.14) in (3.13) it results

$$n_1 \cdot \sin(A_1 - r_1) = n_2 \cdot \sin A_2 \qquad (3.16)$$

or

$$n_1 \cdot (\sin A_1 \cdot \cos r_1 - \sin r_1 \cdot \cos A_1) = n_2 \cdot \sin A_2 \qquad (3.17)$$

Because of (3.12) and (3.15) it results that

$$\sin r_1 = \frac{1}{2n_1} \qquad (3.18)$$

and

$$\cos r_1 = \frac{1}{2n_1}\sqrt{4n_1^2 - 1} \qquad (3.19)$$

Putting together the last three relations it results

$$\sqrt{4n_1^2 - 1} = \frac{2n_2 \cdot \sin A_2 + \cos A_1}{\sin A_1} \qquad (3.20)$$

Because

$\hat{A}_1 = 60°$

and

$\hat{A}_2 = 30°$

relation (3.21) can be written as

$$\sqrt{4n_1^2 - 1} = \frac{2n_2 + 1}{\sqrt{3}} \qquad (3.21)$$

or

$$3 \cdot n_1^2 = 1 + n_2 + n_2^2 \qquad (3.22)$$

Considering the relations (3.1), (3.2) and (3.23) and operating all calculus it results:

$$\lambda^4 \cdot \left(3a_1^2 - a_2^2 - a_2 - 1\right) + \left(6a_1 b_1 - b_2 - 2a_2 b_2\right) \cdot \lambda^2 + 3b_1^2 - b_2^2 = 0 \qquad (3.23)$$

Solving the equation (3.24) one determine the wavelength λ of the ray that enter the prisms system having the direction parallel with DC and emerges the prism system having the direction again parallel with DC. That is

$$\lambda = 1194\, nm \qquad (3.24)$$

or

$$\lambda \cong 1{,}2\, \mu m \qquad (3.25)$$

The relation (3.26) represents the answer of question d.

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