Solution of problem 5:

Theoretical considerations:

a) The acceleration of the center of mass of the cylinder is $a = \frac{2 \cdot s}{t^2}$ (1)

b) Let a_m be the acceleration of the masses m and T the sum of the tensions in the two strings, then

$$T = m \cdot g - m \cdot a_m \quad (2)$$

The acceleration a of the center of mass of the cylinder is determined by the resultant force of the string-tension T and the force of interaction F between cylinder and the horizontal plane.

$$M \cdot a = T - F \quad (3)$$

If the cylinder rotates through an angle θ the mass m moves a distance x_m.
It holds

$$x_m = (R + r) \cdot \theta$$

$$a_m = (R + r) \cdot \frac{a}{R} \quad (4)$$

From (2), (3) and (4) follows $F = mg - \left[M + m \cdot \left(1 + \frac{r}{R} \right) \right] \cdot a$. (5)

c) From the experimental data we see that for small r_i the forces $M \cdot a$ and T are in opposite direction and that they are in the same direction for large r_i .

For small values of r the torque produced by the string-tensions is not large enough to provide the angular acceleration required to prevent slipping. The interaction force between cylinder and plane acts into the direction opposite to the motion of the center of mass and thereby delivers an additional torque.

For large values of r the torque produced by string-tension is too large and the interaction force has such a direction that an opposed torque is produced.

From the rotary-impulse theorem we find

$$T \cdot r + F \cdot R = I \cdot \ddot{\theta} = I \cdot \frac{a}{R},$$

where I is the moment of inertia of the cylinder.

With (3) and (5) you may eliminate T and a from this equation. If the moment of inertia of the cylinder is taken as $I = \frac{1}{2} \cdot M \cdot R^2$ (neglecting the step-up cones) we find after some arithmetical transformations

$$F = mg \cdot \frac{1 - 2 \cdot \frac{r}{R}}{3 + 2 \cdot \frac{m}{M} \cdot \left(1 + \frac{r}{R} \right)^2}.$$

For $r = 0 \rightarrow \quad F = \dfrac{m \cdot g}{3 + 2 \cdot \frac{m}{M}} > 0.$

For $r = R \Rightarrow \quad F = \dfrac{-m \cdot g}{3 + 8 \cdot \frac{m}{M}} < 0.$

Because $\frac{m}{M} \ll 1$ it is approximately $F = \frac{1}{3} m \cdot g - \frac{2}{3} \cdot \frac{r}{R}$.

That means: the dependence of F from r is approximately linear. F will be zero if

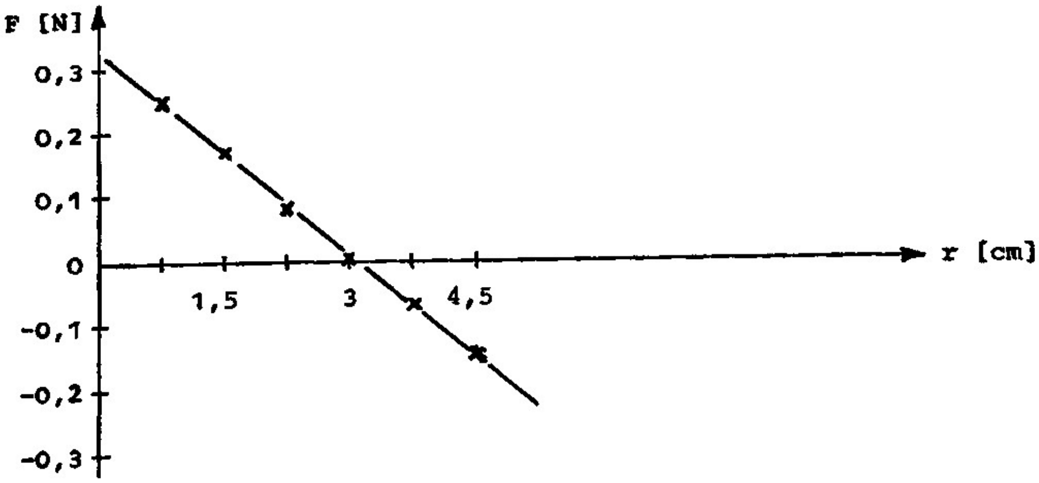
$$\frac{r}{R} = \frac{m \cdot g}{2} .$$

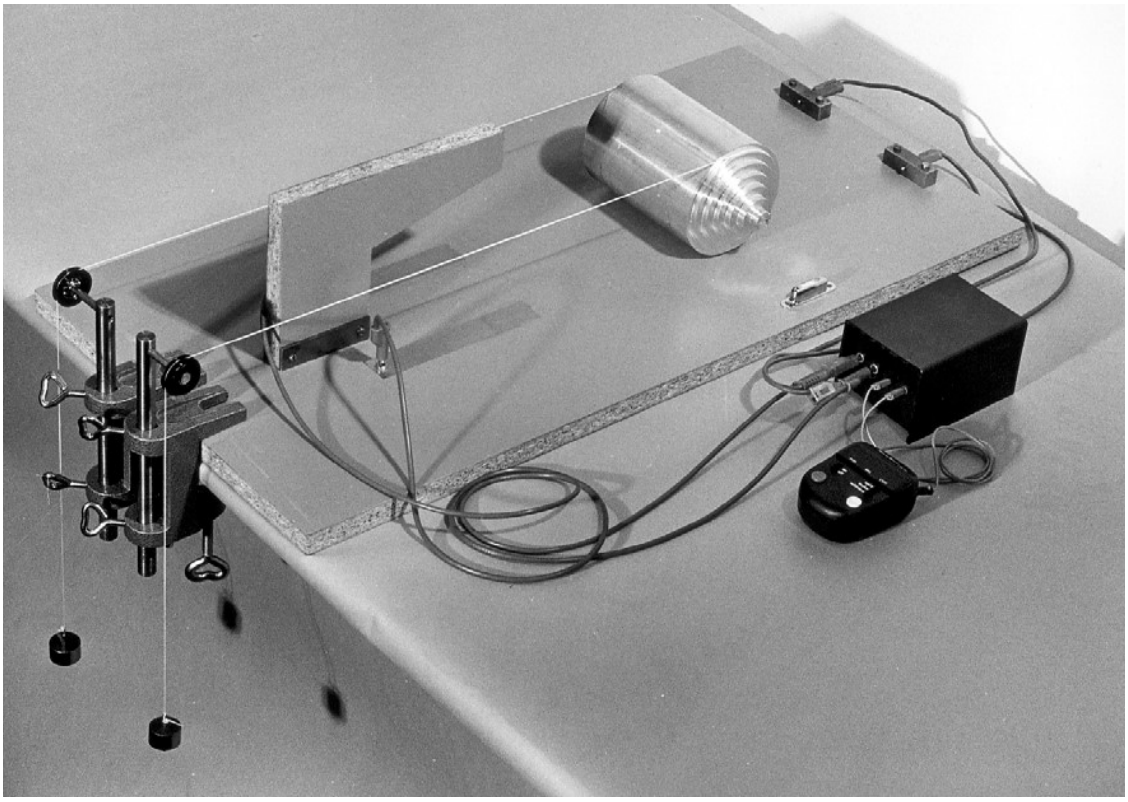
Experimental results:

$$s = L - (2 \cdot R \cdot D + D^2)^{\frac{1}{2}} - (2 \cdot R \cdot d - d^2)^{\frac{1}{2}}$$

$$s = L - 4.5 \text{ cm} = 39.2 \text{ cm} - 4.5 \text{ cm} = 34.7 \text{ cm}$$

r [cm]	t [s]			$\overline{t}$ [s]	a [m/s²]	F [N]
0.75	1.81	1.82	1.82	1.816	0.211	0.266
1.50	1.71	1.72	1.73	1.720	0.235	0.181
2.25	1.63	1.63	1.64	1.633	0.261	0.090
3.00	1.56	1.56	1.57	1.563	0.284	0.004
3.75	1.51	1.51	1.52	1.513	0.304	- 0.066
4.50	1.46	1.46	1.46	1.456	0.328	- 0.154





Grading schemes

Theoretical problems

Problem 1: Fluorescent lamp	pts.
Part a	2
Part b	1
Part c	1
Part d	1
Part e	1
Part f	1
Part g	2
Part h	1
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Problem 2: Oscillating coat hanger	pts.
equation (1)	1,5
equation (2)	1,5
equation (4)	3
equation (5)	2
numerical value for T	1
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Problem 3: Hot-air-balloon	pts.
Part a	3
Part b	2
Part c	3
Part d	2
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Experimental problems

Problem 4: Lens experiment	pts.
correct description of experimental prodedure	1
selection of magnification one	0.5
parallaxe for verifying his magnification	1
$f_L = g = b$ with derivation	1
several measurements with suitable averaging or other determination of error interval	1
taking into account the lens thickness and computing f_L, including the error	0.5
idea of water lens	0.5
theory of lens combination	1
measurements of f'	0.5
calculation of n and correct result	1
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Problem 5: Motion of a rolling cylinder	pts.
Adjustment mentioned of strings a) horizontally and b) in direction of motion	0.5
Indication that angle offset of strings enters the formula for the acting force only quadratically, i.e. by its cosine	0.5
Explanation that with non-horizontal position, the force $m \cdot g$ is to be replaced by $m \cdot g \pm M \cdot g \cdot \sin\alpha$	1.0
Determination of the running length according for formula $s = L - \left(2 \cdot R \cdot D + D^2\right)^{1/2} - \left(2 \cdot R \cdot d + d^2\right)^{1/2}$ including correct numerical result	1.0
Reliable data for rolling time	1.0
accompanied by reasonable error estimate	0.5
Numerical evaluation of the F_i	0.5
Correct plot of $F_i\ (v_i)$	0.5
Qualitative interpretation of the result by intuitive consideration of the limiting cases $r = 0$ and $r = R$	1.0
Indication of a quantitative, theoretical interpretation using the concept of moment of inertia	1.0
Knowledge and application of the formula $a = 2\,s / t^2$	0.5
Force equation for small mass and tension of the string $m \cdot (g - a_m) = T$	1.0
Connection of tension, acceleration of cylinder and reaction force $T - F = M \cdot a$	1.0
Connection between rotary and translatory motion $x_m = (R + r) \cdot \theta$	0.5
$a_m = (1 + r/R) \cdot a$	0.5
Final formula for the reaction force $F = m \cdot g - \left(M + m \cdot (1 + r/R)\right) \cdot a$	1.0
If final formulae are given correctly, the knowledge for preceding equations must be assumed and is graded accordingly.	
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