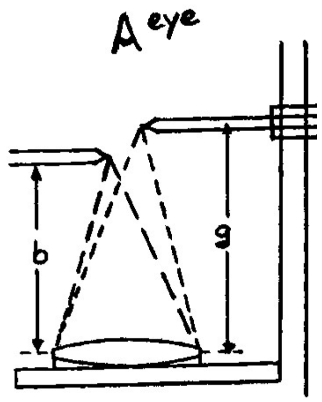
Solution of problem 4:

a) For the determination of f_L , place the lens on the mirror and with the clamp fix the pencil to the supporting base. Lens and mirror are then moved around until the vertically downward looking eye sees the pencil and its image side by side.



In order to have object and image in focus at the same time, they must be placed at an equal distance to the eye. In this case object distance and image distance are the same and the magnification factor is 1 .

It may be proved quite accurately, whether magnification 1 has in fact been obtained, if one concentrates on parallatical shifts between object and image when moving the eye: only when the distances are equal do the pencil-tips point at each other all the time.

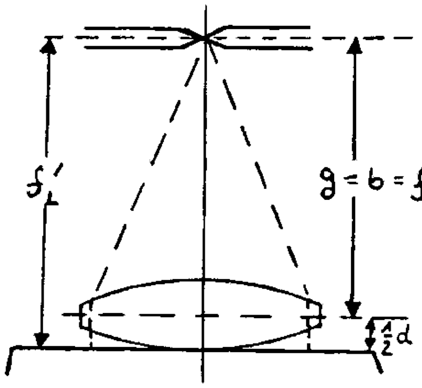
The light rays pass the lens twice because they are reflected by the mirror. Therefore the optical mapping under consideration corresponds to a mapping with two lenses placed directly one after another:

$$\frac{1}{g}+\frac{1}{b}=\frac{1}{f}, \quad \text{where} \quad \frac{1}{f}=\frac{1}{f_L}+\frac{1}{f_L}$$

i.e. the effective focal length f is just half the focal length of the lens. Thus we find for magnification 1:

$$g=b \quad \text{and} \quad \frac{2}{g}=\frac{2}{f_L} \quad \text{i.e.} \quad f_L=g.$$

A different derivation of $f_L = g = b$: For a mapping of magnification 1 the light rays emerging from a point on the optical axis are reflected into themselves. Therefore these rays have to hit the mirror at right angle and so the object distance g equals the focal length f_L of the lens in this case.



The distance between pencil point and mirror has to be determined with an accuracy, which enables one to state f_L with a maximum error of $\pm 1\ \%$. This is accomplished either by averaging several measurements or by stating an uncertainty interval, which is found through the appearance of parallaxe.

Half the thickness of the lens has to be subtracted from the distance between pencil-point and mirror.

$$f_L = f_L' - \frac{1}{2}d, \quad d = 3.0 \pm 0.5\ \text{mm}$$

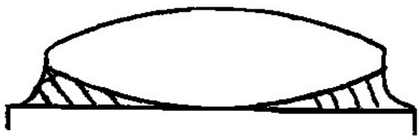
The nominal value of the focal length of the lens is $f_L = 30$ cm. However, the actual focal length of the single lenses spread considerably. Each lens was measured separately, so the individual result of the student can be compared with the exact value.

b) The refractive index n of the lens material can be evaluated from the equation

$$\frac{1}{f_L} = (n-1)\cdot\frac{2}{r}$$

if the focal length f_L and the curvature radius r of the symmetric biconvex lens are known. f_L was determined in part a) of this problem.

The still unknown curvature radius r of the symmetric biconvex lens is found in the following way: If one pours some water onto the mirror and places the lens in the water, one gets a plane-concave water lens, which has one curvature radius equalling the glass lens' radius and the other radius is ∞ .



Because the refractive index of water is known in this case, one can evaluate the curvature radius through the formula above, where $r_1 = -r$ and $r_2 = \infty$:

$$-\frac{1}{f_w} = (n_w - 1) \cdot \frac{1}{r}.$$

Only the focal length f' of the combination of lenses is directly measured, for which we have

$$\frac{1}{f'} = \frac{1}{f_L} + \frac{1}{f_w}.$$

This focal length is to be determined by a mapping of magnification 1 as above.

Then the focal length of the water lens is $\frac{1}{f_w} = \frac{1}{f'} - \frac{1}{f_L}$

and one has the curvature radius $r = -(n_w - 1) \cdot f_w$.

Now the refractive index of the lens is determined by $n = \frac{r}{2 \cdot f_L} + 1$

with the known values of f_L and r, or, if one wants to express n explicitly through the measured quantities: $n = \frac{f' \cdot (n_w - 1)}{2 \cdot (f' - f_L)} + 1.$

The nominal values are: $f' = 43.9$ cm, $f_w = -94.5$ cm, $r = 31.2$ cm, $n = 1.52$.