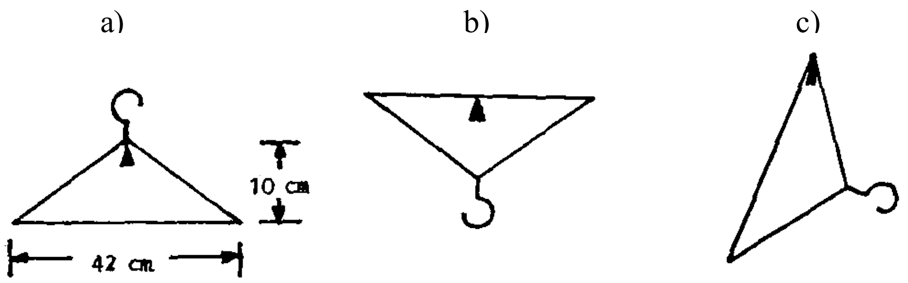
horizontal. The other two sides have equal length. The period of oscillation is the same in all cases.

What is the location of the center of mass, and how long is the period?

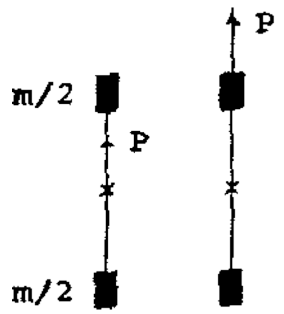


The figure does not contain any information beyond the dimensions given. Nothing is known, e.g., concerning the detailed distribution of mass.

Solution of problem 2

First method:

The motions of a rigid body in a plane correspond to the motion of two equal point masses connected by a rigid massless rod. The moment of inertia then determines their distance. Because of the equilibrium position a) the center of mass is on the perpendicular bipartition of the long side of the coat hanger. If one imagines the equivalent masses and the supporting point P being arranged in a straight line in each case, only two positions of P yield the same period of oscillation (see sketch). One can understand this by considering the limiting cases: 1. both supporting points in the upper mass and 2. one point in the center of mass and the other infinitely high above. Between these extremes the period of oscillation grows continuously. The supporting point placed in the corner of the long side c) has the largest distance from the center of mass, and therefore this point lies outside the two point masses. The two other supporting points a), b) then have to be placed symmetrically to the center of mass between the two point masses, i.e., the center of mass bisects the perpendicular bipartition. One knows of the reversible pendulum that for every supporting point of the physical pendulum it generally has a second supporting point of the pendulum rotated by 180°, with the same period of oscillation but at a different distance from the center of mass. The



section between the two supporting points equals the length of the corresponding mathematical pendulum. Therefore the period of oscillation is obtained through the corresponding length of the pendulum $s_b + s_c$, where $s_b = 5$ cm and $s_c = \sqrt{5^2 + 21^2}$ cm, to be $T = 1.03$ s .

Second method:

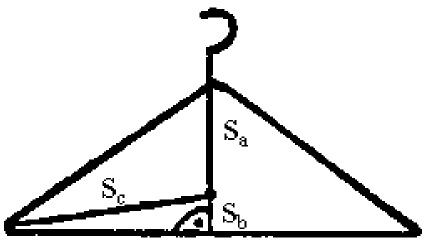
Let s denote the distance between the supporting point and the center of mass, m the mass itself and θ the moment of inertia referring to the supporting point. Then we have the period of oscillation T :

$$T = 2\pi\sqrt{\frac{\theta}{m \cdot g \cdot s}} \quad , \tag{1}$$

where g is the acceleration of gravity, $g = 9.81$ m/s^2. Here θ can be obtained from the moment of inertia θ_o related to the center of mass:

$$\theta = \theta_0 + m \cdot s^2 \tag{2}$$

Because of the symmetrical position in case a) the center of mass is to be found on the perpendicular bisection above the long side. Now (1) and (2) yield



$$\theta_0 + m \cdot s^2 = \left(\frac{T}{2 \cdot \pi}\right)^2 \cdot m \cdot g \cdot s \quad \text{for} \quad s = s_a,\ s_b \text{ and } s_c. \tag{3}$$

because all periods of oscillation are the same. This quadratic equation has only two different solutions at most. Therefore at least two of the three distances are equal. Because of $s_c > 21$ cm $> s_a + s_b$, only s_a and s_b can equal each other. Thus we have

$$s_a = 5 \text{ cm} \tag{4}$$

The moment of inertia θ_0 is eliminated through (3):

$$m \cdot (s_c^{\,2} - s_a^{\,2}) = \left(\frac{T}{2 \cdot \pi}\right)^2 \cdot m \cdot g \cdot (s_c - s_a)$$

and we have $$T = 2 \cdot \pi\sqrt{\frac{s_c + s_a}{g}} \tag{5}$$

with the numerical value $T = 1.03\,\mathrm{s}$,

which has been rounded off after two decimals because of the accuracy of g.

Third method:

This solution is identical to the previous one up to equation (2).

From (1) and (2) we generally have for equal periods of oscillation $T_1 = T_2$:

$$\frac{\theta_0 + m \cdot s_1^2}{m \cdot g \cdot s_1} = \frac{\theta_0 + m \cdot s_2^2}{m \cdot g \cdot s_2}$$

and therefore $s_2 \cdot \left(\theta_0 + m \cdot s_1^2\right) = s_1 \cdot \left(\theta_0 + m \cdot s_2^2\right)$

or $$\left(s_2 - s_1\right) \cdot \left(\theta_0 - m \cdot s_1\, s_2\right) = 0 \qquad (6)$$

The solution of (6) includes two possibilities: $s_1 = s_2$ or $s_1 \cdot s_2 = \dfrac{\theta_0}{m}$

Let $2{\cdot}a$ be the length of the long side and b the height of the coat hanger. Because of $T_b = T_c$ we then have either $s_b = s_c$ or $s_b \cdot s_c = \dfrac{\theta_0}{m}$, where $s_c = \sqrt{s_b^{\,2} + a^2}$,

which excludes the first possibility. Thus $$s_b \cdot s_c = \frac{\theta_0}{m} . \qquad (7)$$

For $T_a = T_b$ the case $s_a \cdot s_b = \dfrac{\theta_0}{m}$ is excluded because of eq. (7), for we have

$$s_a \cdot s_b < s_c \cdot s_b = \frac{\theta_0}{m} .$$

Hence $s_a = s_b = \dfrac{1}{2} b, \quad s_c = \sqrt{\dfrac{1}{4} b^2 + a^2}$

and $$T = 2 \cdot \pi \sqrt{\frac{\frac{\theta_0}{m} + s_b^{\,2}}{g \cdot s_b}} = 2\pi \sqrt{\frac{s_b \cdot s_c + s_b^{\,2}}{g \cdot s_b}}$$

The numerical calculation yields the value $T = 1.03\,\mathrm{s}$.