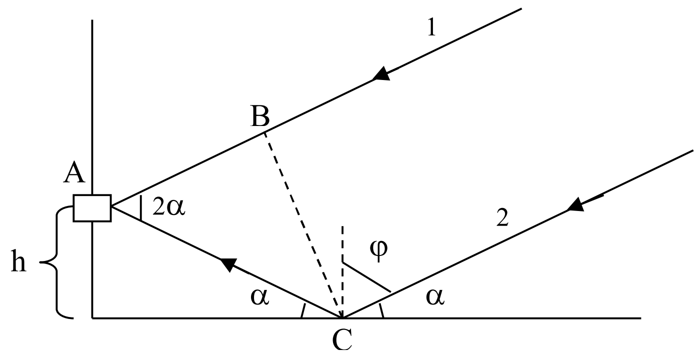
Solution of the Theoretical Problem 3

1) The signal, registered by the detector A, is result of the interference of two rays: the ray 1, incident directly from the star and the ray 2, reflected from the sea surface (see the figure).



The phase of the second ray is shifted by π due to the reflection by a medium of larger refractive index. Therefore, the phase difference between the two rays is:

$$\Delta = AC + \frac{\lambda}{2} - AB = \frac{h}{\sin\alpha} + \frac{\lambda}{2} - \left(\frac{h}{\sin\alpha}\right)\cos(2\alpha) =$$

$$= \frac{\lambda}{2} + \frac{h}{\sin\alpha}[1-\cos(2\alpha)] = \frac{\lambda}{2} + 2h\sin\alpha \qquad (1)$$

The condition for an interference maximum is:

$$\frac{\lambda}{2} + 2h\sin\alpha_{\max} = k\lambda\text{, or}$$

$$\sin\alpha_{\max} = (k-\frac{1}{2})\frac{\lambda}{2h} = (2k-1)\frac{\lambda}{4h}, \qquad (2)$$

where k = 1,2,3,…,19. (the difference of the optical paths cannot exceed $2h$, therefore k cannot exceed 19).

The condition for an interference minimum is:

$$\frac{\lambda}{2} + 2h\sin\alpha_{\max} = (2k+1)\frac{\lambda}{2}\text{, or}$$

$$\sin\alpha_{\min} = \frac{k\lambda}{2h} \qquad (3)$$

where k = 1,2,3,…,19.

2) Just after the rise of the star the angular height α is zero, therefore the condition for an interference minimum is satisfied. By this reason just after the rise of the star, the signal will increase.

3) If the condition for an interference maximum is satisfied, the intensity of the electric field is a sum of the intensities of the direct ray E_i and the reflected ray E_r, respectively: $E_{\max} = E_i + E_r$.

Because $E_r = E_i\frac{n-\cos\varphi}{n+\cos\varphi}$, then $E_{\max} = E_i\left(1+\frac{n-\cos\varphi_{\max}}{n+\cos\varphi_{\max}}\right)$.

From the figure it is seen that $\varphi_{\max} = \frac{\pi}{2} - \alpha_{\max}$, we obtain

$$E_{\max} = E_i\left(1+\frac{n-\sin\alpha_{\max}}{n+\sin\alpha_{\max}}\right) = E_i\frac{2n}{n+\sin(2\alpha_{\max})}. \qquad (4)$$

At the interference minimum, the resulting intensity is:

$$E_{\min} = E_i - E_r = E_i\frac{2\sin\alpha_{\min}}{n+\sin\alpha_{\min}}. \qquad (5)$$

The intensity I of the signal is proportional to the square of the intensity of the electric field E, therefore the ratio of the intensities of the consecutive maxima and minima is:

$$\frac{I_{\max}}{I_{\min}} = \left(\frac{E_{\max}}{E_{\min}}\right)^2 = \frac{n^2}{\sin^2\alpha_{\min}}\frac{(n+\sin\alpha_{\min})^2}{(n+\sin\alpha_{\max})^2}. \qquad (6)$$

Using the eqs. (2) and (3), the eq. (6) can be transformed into the following form:

$$\frac{I_{\max}}{I_{\min}} = \frac{4n^2h^2}{k^2\lambda^2}\left[\frac{n+k\dfrac{\lambda}{2h}}{n+(2k-1)\dfrac{\lambda}{4h}}\right]^2. \qquad (7)$$

Using this general formula, we can determine the ratio for the first maximum (k =1) and the next minimum:

$$\frac{I_{\max}}{I_{\min}} = \frac{4n^2h^2}{\lambda^2}\left(\frac{n+\dfrac{\lambda}{2h}}{n+\dfrac{\lambda}{4h}}\right)^2 = 3.10^4$$

4) Using that $n \gg \dfrac{\lambda}{2h}$, from the eq. (7) follows :

$$\frac{I_{\max}}{I_{\min}} \approx \frac{4n^2h^2}{k^2\lambda^2}.$$

So, with the rising of the star the ratio of the intensities of the consecutive maxima and minima decreases.