Problem 2.

A sample and weights are affected by the Archimede's buoyancy force of either dry or humid air in the first and second cases, respectively. The difference in the scale indication ΔF is determined by the change of difference of these forces.

The difference of Archimede's buoyancy forces in dry air:

$$\Delta F_1 = \Delta V \rho_a^{'} g$$

Whereas in humid air it is:

$$\Delta F_2 = \Delta V \rho_a^{''} g$$

where ΔV - the difference in volumes between the sample and the weights, and $\rho_a^{'}$ and $\rho_a^{''}$ - densities of dry and humid air, respectively.

Then the difference in the scale indications ΔF could be written as follows:

$$\Delta F = \Delta F_1 - \Delta F_2 = \Delta V g \left(\rho_a^{'} - \rho_a^{''} \right) \qquad (1)$$

According to the problem conditions this difference should be distinguished, i.e.
$\Delta F \geq m_0 g$ or $\Delta V g \left(\rho_a^{'} - \rho_a^{''} \right) \geq m_0$, wherefrom

$$\Delta V \geq \frac{m_0}{\rho_a^{'} - \rho_a^{''}} . \qquad (2)$$

The difference in volumes between the aluminum sample and brass weights can be found from the equation

$$\Delta V = \frac{m}{\rho_1} - \frac{m}{\rho_2} = m \left(\frac{\rho_2 - \rho_1}{\rho_1 \rho_2} \right) , \qquad (3)$$

where m is the sought mass of the sample. From expressions (2) and (3) we obtain

$$m = \Delta V \left(\frac{\rho_1 \rho_2}{\rho_2 - \rho_1} \right) \geq \frac{m_0}{\rho_a^{'} - \rho_a^{''}} \left(\frac{\rho_1 \rho_2}{\rho_2 - \rho_1} \right) . \qquad (4)$$

To find the mass m of the sample one has to determine the difference $\left(\rho_a^{'} - \rho_a^{''} \right)$.

With the general pressure being equal, in the second case, some part of dry air is replaced by vapor:

$$\rho_a^{'} - \rho_a^{''} = \frac{\Delta m_a}{V} - \frac{\Delta m_v}{V} .$$

Changes of mass of air Δm_a and vapor Δm_v can be found from the ideal-gas equation of state

$$\Delta m_a = \frac{P_a V M_a}{RT} , \quad \Delta m_v = \frac{P_v V M_v}{RT} ,$$

wherefrom we obtain

$$\rho_a^{'} - \rho_a^{''} = \frac{P_a \left(M_a - M_v \right)}{RT} . \qquad (5)$$

From equations (4) and (5) we obtain

$$m \geq \frac{m_0 RT}{P_a(M_a - M_v)}\left(\frac{\rho_1 \rho_2}{\rho_2 - \rho_1}\right). \qquad (6)$$

The substitution of numerical values gives the answer: $m \geq 0.0432$ kg ≈ 43 g.

Note. When we wrote down expression (3), we considered the sample mass be equal to the weights' mass, at the same time allowing for a small error.

One may choose another way of solving this problem. Let us calculate the change of Archimede's force by the change of the air average molar mass.

In dry air the condition of the balance between the sample and weights could be written down in the form of

$$\left(\rho_1 - \frac{M_a P}{RT}\right)V_1 = \left(\rho_2 - \frac{M_a P}{RT}\right)V_2 \ . \qquad (7)$$

In humid air its molar mass is equal to

$$M = M_a \frac{P_a}{P} + M_v \frac{P - P_a}{P}, \qquad (8)$$

whereas the condition of finding the scale error could be written in the form

$$\left(\rho_1 - \frac{M_a P}{RT}\right)V_1 - \left(\rho_2 - \frac{M_a P}{RT}\right)V_2 \geq m_0. \qquad (9)$$

From expressions (7) –(9) one can get a more precise answer

$$m \geq \frac{m_0 RT \rho_1 \rho_2 - M_a P_a}{(M_a - M_v)(\rho_2 - \rho_1)P_a} \ . \qquad (10)$$

Since $M_a P_a << m_0 \rho_1 \rho_2 RT$, then both expressions (6) and (10) lead practically to the same quantitative result, i.e. m ≥ 43 g.