a) Which processes run between the points 0–1, 2–3, 4–1, 1–0?

b) Determine the pressure and the temperature in the states 1, 2, 3 and 4.

c) Find the thermal efficiency of the cycle.

d) Discuss obtained results. Are they realistic?

Solution: a) The description of the processes between particular points is the following:

0–1 :	intake stroke	isobaric and isothermal process
1–2 :	compression of the mixture	adiabatic process
2–3 :	mixture ignition	isochoric process
3–4 :	expansion of the exhaust gas	adiabatic process
4–1 :	exhaust	isochoric process
1–0 :	exhaust	isobaric process

Let us denote the initial volume of the cylinder before induction at the point 0 by V_1, after induction at the point 1 by V_2 and the temperatures at the particular points by T_0, T_1, T_2, T_3 and T_4.

b) The equations for particular processes are as follows.

0–1 : The fuel-air mixture is drawn into the cylinder at the temperature of $T_0 = T_1 = 300$ K and a pressure of $p_0 = p_1 = 0.10$ MPa.

1–2 : Since the compression is very fast, one can suppose the process to be adiabatic. Hence:

$$p_1 V_2^\kappa = p_2 V_1^\kappa \quad \text{and} \quad \frac{p_1 V_2}{T_1} = \frac{p_2 V_1}{T_2}\,.$$

From the first equation one obtains

$$p_2 = p_1 \left(\frac{V_2}{V_1}\right)^\kappa = p_1 \varepsilon^\kappa$$

and by the dividing of both equations we arrive after a straightforward calculation at

$$T_1 V_2^{\kappa-1} = T_2 V_1^{\kappa-1}\,, \quad T_2 = T_1 \left(\frac{V_2}{V_1}\right)^{\kappa-1} = T_1 \varepsilon^{\kappa-1}\,.$$

For given values $\kappa = 1.40$, $\varepsilon = 9.5$, $p_1 = 0.10$ MPa, $T_1 = 300$ K we have $p_2 = 2.34$ MPa and $T_2 = 738$ K ($t_2 = 465\,^\circ$C).

2–3 : Because the process is isochoric and $p_3 = 2p_2$ holds true, we can write

$$\frac{p_3}{p_2} = \frac{T_3}{T_2}\,, \quad \text{which implies} \quad T_3 = T_2 \frac{p_3}{p_2} = 2T_2\,.$$

Numerically, $p_3 = 4.68$ MPa, $T_3 = 1476$ K ($t_3 = 1203\,^\circ$C).

3–4 : The expansion is adiabatic, therefore

$$p_3 V_1^\kappa = p_4 V_2^\kappa\,, \quad \frac{p_3 V_1}{T_3} = \frac{p_4 V_2}{T_4}\,.$$

The first equation gives

$$p_4 = p_3 \left(\frac{V_1}{V_2}\right)^\kappa = 2p_2\varepsilon^{-\kappa} = 2p_1$$

and by dividing we get

$$T_3 V_1^{\kappa-1} = T_4 V_2^{\kappa-1}\,.$$

Consequently,

$$T_4 = T_3 \varepsilon^{1-\kappa} = 2T_2\varepsilon^{1-\kappa} = 2T_1\,.$$

Numerical results: $p_4 = 0.20$ MPa, $T_3 = 600$ K ($t_3 = 327\,^\circ$C).

4–1 : The process is isochoric. Denoting the temperature by T_1' we can write

$$\frac{p_4}{p_1} = \frac{T_4}{T_1'}\,,$$

which yields

$$T_1' = T_4 \frac{p_1}{p_4} = \frac{T_4}{2} = T_1\,.$$

We have thus obtained the correct result $T_1' = T_1$. Numerically, $p_1 = 0.10$ MPa, $T_1' = 300$ K.

c) Thermal efficiency of the engine is defined as the proportion of the heat supplied that is converted to net work. The exhaust gas does work on the piston during the expansion 3–4, on the other hand, the work is done on the mixture during the compression 1–2. No work is done by/on the gas during the processes 2–3 and 4–1. The heat is supplied to the gas during the process 2–3.

The net work done by 1 mol of the gas is

$$W = \frac{R}{\kappa - 1}(T_1 - T_2) + \frac{R}{\kappa - 1}(T_3 - T_4) = \frac{R}{\kappa - 1}(T_1 - T_2 + T_3 - T_4)$$

and the heat supplied to the gas is

$$Q_{23} = C_V(T_3 - T_2)\,.$$

Hence, we have for thermal efficiency

$$\eta = \frac{W}{Q_{23}} = \frac{R}{(\kappa - 1)C_V}\,\frac{T_1 - T_2 + T_3 - T_4}{T_3 - T_2}\,.$$

Since

$$\frac{R}{(\kappa - 1)C_V} = \frac{C_p - C_V}{(\kappa - 1)C_V} = \frac{\kappa - 1}{\kappa - 1} = 1\,,$$

we obtain

$$\eta = 1 - \frac{T_4 - T_1}{T_3 - T_2} = 1 - \frac{T_1}{T_2} = 1 - \varepsilon^{1-\kappa}\,.$$

Numerically, $\eta = 1 - 300/738 = 1 - 0.407$, $\eta = 59,3\%$.

d) Actually, the real pV-diagram of the cycle is smooth, without the sharp angles. Since the gas is not ideal, the real efficiency would be lower than the calculated one.

Problem 2. Dipping the frame in a soap solution, the soap forms a rectangle film of length b and height h. White light falls on the film at an angle α (measured with respect to the normal direction). The reflected light displays a green color of wavelength λ_0.

a) Find out if it is possible to determine the mass of the soap film using the laboratory scales which has calibration accuracy of 0.1 mg.

b) What color does the thinnest possible soap film display being seen from the perpendicular direction? Derive the related equations.

Constants and given data: relative refractive index $n = 1.33$, the wavelength of the reflected green light $\lambda_0 = 500$ nm, $\alpha = 30^\circ$, $b = 0.020$ m, $h = 0.030$ m, density $\varrho = 1000$ kg m^{-3}.