Remark: $r_i > 0$ means that the central curvature point M_i is on the right side of the aerial vertex S_i, $r_i < 0$ means that the central curvature point M_i is on the left side of the aerial vertex S_i (i = 1,2).

For some special applications it is required, that the focal length is independent from the wavelength.

a) For how many different wavelengths can the same focal length be achieved?

b) Describe a relation between r_i (i = 1,2), d and the refractive index n for which the required wavelength independence can be fulfilled and discuss this relation.
Sketch possible shapes of lenses and mark the central curvature points M_1 and M_2.

c) Prove that for a given planconvex lens a specific focal length can be achieved by only one wavelength.

d) State possible parameters of the thick lens for two further cases in which a certain focal length can be realized for one wavelength only. Take into account the physical and the geometrical circumstances.

Solution of problem 2:

a) The refractive index n is a function of the wavelength λ, i.e. $n = n(\lambda)$. According to the given formula for the focal length f (see above) which for a given f yields to an equation quadratic in n there are at most two different wavelengths (indices of refraction) for the same focal length.

b) If the focal length is the same for two different wavelengths, then the equation

$$f(\lambda_1) = f(\lambda_2) \quad or \quad f(n_1) = f(n_2) \tag{1}$$

holds. Using the given equation for the focal length it follows from equation (1):

$$\frac{n_1 r_1 r_2}{(n_1 - 1)\left[n_1(r_2 - r_1) + d(n_1 - 1)\right]} = \frac{n_2 r_1 r_2}{(n_2 - 1)\left[n_2(r_2 - r_1) + d(n_2 - 1)\right]}$$

Algebraic calculations lead to:

$$r_1 - r_2 = d \cdot \left(1 - \frac{1}{n_1 n_2}\right) \tag{2}.$$

If the values of the radii r_1, r_2 and the thickness satisfy this condition the focal length will be the same for two wavelengths (indices of refraction). The parameters in this equation are subject to some physical restrictions: The indices of refraction are greater than 1 and the thickness of the lens is greater than 0 m. Therefore, from equation (2) the relation

$$d > r_1 - r_2 > 0 \tag{3}$$

Solutions of equation (5) are:

$$n_{1,2} = -\frac{B}{2 \cdot A} \pm \sqrt{\frac{B^2}{4 \cdot A^2} - \frac{C}{A}} \qquad (6).$$

Equation (5) has only one physical correct solution, if...

I) A = 0 (i.e., the coefficient of n^2 in equation (5) vanishes)

In this case the following relationships exists:

$$r_1 - r_2 = d \qquad (7),$$

$$n = \frac{f \cdot d}{f \cdot d + r_1 \cdot r_2} > 1 \qquad (8).$$

II) B = 0 (i.e. the coefficient of n in equation (5) vanishes)

In this case the equation has a positive and a negative solution. Only the positve solution makes sense from the physical point of view. It is:

$$f \cdot (r_2 - r_1) + 2 \cdot f \cdot d + r_1 \cdot r_2 = 0 \qquad (9),$$

$$n^2 = -\frac{C}{A} = -\frac{d}{(r_2 - r_1 + d)} > 1 \qquad (10),$$

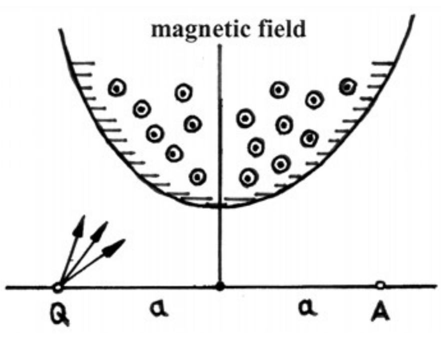
III) $B^2 = 4$ AC

In this case two identical real solutions exist. It is:

$$\left[f \cdot (r_2 - r_1) + 2 \cdot f \cdot d + r_1 \cdot r_2\right]^2 = 4 \cdot (r_2 - r_1 + d) \cdot f^2 \cdot d \qquad (11),$$

$$n = -\frac{B}{2 \cdot A} = \frac{f \cdot (r_2 - r_1) + 2 \cdot f \cdot d + r_1 \cdot r_2}{2 \cdot f (r_2 - r_1 + d)} > 1 \qquad (12).$$

Theoretical problem 3: “Ions in a magnetic field”



A beam of positive ions (charge +e) of the same and constant mass m spread from point Q in different directions in the plane of paper (see figure[2]). The ions were accelerated by a voltage U. They are deflected in a uniform magnetic field B that is perpendicular to the plane of paper. The boundaries of the magnetic field are made in a way that the initially diverging ions are focussed in point A ($\overline{QA} = 2 \cdot a$). The trajectories of the ions are symmetric to the middle perpendicular on $\overline{QA}$.

[2] Remark: This illustrative figure was <u>not</u> part of the original problem formulation.