Solution of problem 1:

a) $\omega = 0$:

The forces in this case are (see figure):
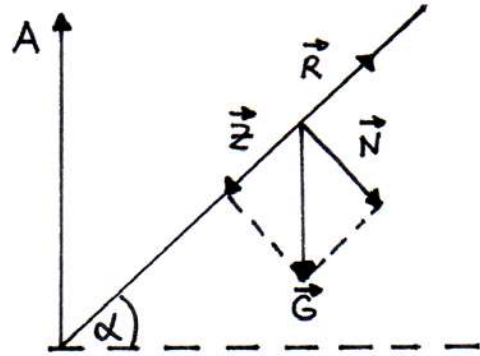
$$\vec{G} = \vec{Z} + \vec{N} = m \cdot \vec{g} \qquad (1),$$

$$\left| \vec{Z} \right| = m \cdot g \cdot \sin\alpha = Z \qquad (2),$$

$$\left| \vec{N} \right| = m \cdot g \cdot \cos\alpha = N \qquad (3),$$

$$\left| \vec{R} \right| = \mu \cdot N = \mu \cdot m \cdot g \cdot \cos\alpha = R \qquad (4).$$

[$\vec{R}$: force of friction]

The body is at rest relative to the rod, if $Z \le R$. According to equations (2) and (4) this is equivalent to $\tan\alpha \le \tan\beta$. That means, the body is at rest relative to the rod for $\alpha \le \beta$ and the body moves along the rod for $\alpha > \beta$.

b) $\omega > 0$:

Two different situations have to be considered: 1. $\alpha > \beta$ and 2. $\alpha \le \beta$.
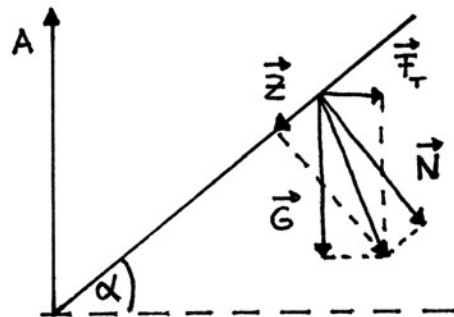
If the rod is moving ($\omega \ne 0$) the forces are $\vec{G} = m \cdot \vec{g}$ and $\left| \vec{F}_r \right| = m \cdot r \cdot \vec{\omega}^2$.

From the parallelogramm of forces (see figure):

$$\vec{Z} + \vec{N} = \vec{G} + \vec{F}_r \qquad (5).$$

The condition of equilibrium is:

$$\left| \vec{Z} \right| = \mu \left| \vec{N} \right| \qquad (6).$$

Case 1: $\vec{Z}$ is oriented downwards, i.e. $g \cdot \sin\alpha > r \cdot \omega^2 \cdot \cos\alpha$.

$$\left| \vec{Z} \right| = m \cdot g \cdot \sin\alpha - m \cdot r \cdot \omega^2 \cdot \cos\alpha \quad \text{and} \quad \left| \vec{N} \right| = m \cdot g \cdot \cos\alpha + m \cdot r \cdot \omega^2 \cdot \sin\alpha$$

Case 2: $\vec{Z}$ is oriented upwards, i.e. $g \cdot \sin\alpha < r \cdot \omega^2 \cdot \cos\alpha$.

$$\left| \vec{Z} \right| = -m \cdot g \cdot \sin\alpha + m \cdot r \cdot \omega^2 \cdot \cos\alpha \quad \text{and} \quad \left| \vec{N} \right| = m \cdot g \cdot \cos\alpha + m \cdot r \cdot \omega^2 \cdot \sin\alpha$$

It follows from the condition of equilibrium equation (6) that

$$\pm \left(g \cdot \sin\alpha - r \cdot \omega^2 \cdot \cos\alpha \right) = \tan\beta \cdot \left(g \cdot \cos\alpha + r \cdot \omega^2 \cdot \sin\alpha \right) \qquad (7).$$

Algebraic manipulation of equation (7) leads to:

$$g \cdot \sin(\alpha - \beta) = r \cdot \omega^2 \cdot \cos(\alpha - \beta) \qquad (8),$$

$$g \cdot \sin(\alpha + \beta) = r \cdot \omega^2 \cdot \cos(\alpha + \beta) \qquad (9).$$

That means,

$$r_{1,2} = \frac{g}{\omega^2} \cdot \tan(\alpha \mp \beta) \qquad (10).$$

The body is at rest relative to the rotating rod in the case $\alpha > \beta$ if the following inequalities hold:

$$r_1 \le r \le r_2 \qquad \text{with } r_1, r_2 > 0 \qquad (11)$$

or

$$L_1 \le L \le L_2 \qquad \text{with } L_1 = r_1 / \cos\alpha \text{ and } L_2 = r_2 / \cos\alpha \qquad (12).$$

The body is at rest relative to the rotating rod in the case $\alpha \le \beta$ if the following inequalities hold:

$$0 \le r \le r_2 \qquad \text{with } r_1 = 0 \text{ (since } r_1 < 0 \text{ is not a physical solution)}, \ r_2 > 0 \qquad (13).$$

Inequality (13) is equivalent to

$$0 \le L \le L_2 \qquad \text{with } L_2 = r_2 / \cos\alpha > 0 \qquad (14).$$

Theoretical problem 2:   "Thick lens"

The focal length f of a thick glass lens in air with refractive index n, radius curvatures r_1, r_2 and

vertex distance d (see figure) is given by: $$f = \frac{n\, r_1 r_2}{(n-1)\left[n(r_2 - r_1) + d(n-1)\right]}$$