Algebraic manipulation of equation (7) leads to:

$$g \cdot \sin(\alpha - \beta) = r \cdot \omega^2 \cdot \cos(\alpha - \beta) \qquad (8),$$

$$g \cdot \sin(\alpha + \beta) = r \cdot \omega^2 \cdot \cos(\alpha + \beta) \qquad (9).$$

That means,

$$r_{1,2} = \frac{g}{\omega^2} \cdot \tan(\alpha \mp \beta) \qquad (10).$$

The body is at rest relative to the rotating rod in the case $\alpha > \beta$ if the following inequalities hold:

$$r_1 \le r \le r_2 \qquad \text{with } r_1, r_2 > 0 \qquad (11)$$

or

$$L_1 \le L \le L_2 \qquad \text{with } L_1 = r_1 / \cos\alpha \text{ and } L_2 = r_2 / \cos\alpha \qquad (12).$$

The body is at rest relative to the rotating rod in the case $\alpha \le \beta$ if the following inequalities hold:

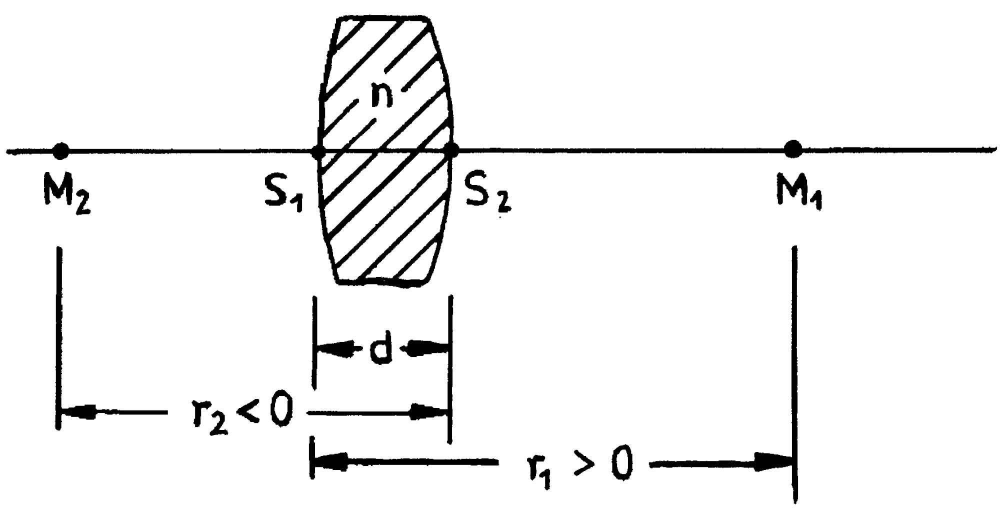
$$0 \le r \le r_2 \qquad \text{with } r_1 = 0 \text{ (since } r_1 < 0 \text{ is not a physical solution), } r_2 > 0 \qquad (13).$$

Inequality (13) is equivalent to

$$0 \le L \le L_2 \qquad \text{with } L_2 = r_2 / \cos\alpha > 0 \qquad (14).$$

Theoretical problem 2: “Thick lens”

The focal length f of a thick glass lens in air with refractive index n, radius curvatures r_1, r_2 and vertex distance d (see figure) is given by: $f = \dfrac{n r_1 r_2}{(n-1)\left[n(r_2 - r_1) + d(n-1)\right]}$



Remark: $r_i > 0$ means that the central curvature point M_i is on the right side of the aerial vertex S_i, $r_i < 0$ means that the central curvature point M_i is on the left side of the aerial vertex S_i ($i = 1,2$).

For some special applications it is required, that the focal length is independent from the wavelength.

a) For how many different wavelengths can the same focal length be achieved?

b) Describe a relation between r_i ($i = 1,2$), d and the refractive index n for which the required wavelength independence can be fulfilled and discuss this relation.

   Sketch possible shapes of lenses and mark the central curvature points M_1 and M_2.

c) Prove that for a given planconvex lens a specific focal length can be achieved by only one wavelength.

d) State possible parameters of the thick lens for two further cases in which a certain focal length can be realized for one wavelength only. Take into account the physical and the geometrical circumstances.

Solution of problem 2:

a) The refractive index n is a function of the wavelength λ, i.e. $n = n(\lambda)$. According to the given formula for the focal length f (see above) which for a given f yields to an equation quadratic in n there are at most two different wavelengths (indices of refraction) for the same focal length.

b) If the focal length is the same for two different wavelengths, then the equation

$$f(\lambda_1) = f(\lambda_2) \quad or \quad f(n_1) = f(n_2) \tag{1}$$

holds. Using the given equation for the focal length it follows from equation (1):

$$\frac{n_1 r_1 r_2}{(n_1 - 1)\left[n_1(r_2 - r_1) + d(n_1 - 1)\right]} = \frac{n_2 r_1 r_2}{(n_2 - 1)\left[n_2(r_2 - r_1) + d(n_2 - 1)\right]}$$

Algebraic calculations lead to:

$$r_1 - r_2 = d \cdot \left(1 - \frac{1}{n_1 n_2}\right) \tag{2}$$

If the values of the radii r_1, r_2 and the thickness satisfy this condition the focal length will be the same for two wavelengths (indices of refraction). The parameters in this equation are subject to some physical restrictions: The indices of refraction are greater than 1 and the thickness of the lens is greater than 0 m. Therefore, from equation (2) the relation

$$d > r_1 - r_2 > 0 \tag{3}$$