***Solution***

Let us construct the device shown in Fig. 6. $B_1$ denotes the membrane transparent for the gas from the cavity, but non-transparent for the air, while $B_2$ denotes the membrane with opposite property: it is transparent for the air but non-transparent for the gas.

Initially the valve $Z_1$ is open and the valve $Z_2$ is closed. In the initial situation, when we keep the piston at rest, the pressure under the piston is equal to $p_0 + p_0$ due to the Dalton law. Let $V_0$ denotes an initial volume of the gas (at pressure $p_0$).

Now we close the valve $Z_1$ and allow the gas in the cylinder to expand. During movement of the piston in the downwards direction we obtain certain work performed by excess pressure inside the cylinder with respect to the atmospheric pressure $p_0$. The partial pressure of the gas in the cylinder will be reduced according to the formula $p = p_0 V_0 / V$, where $V$ denotes volume closed by the piston (isothermal process). Due to the membrane $B_2$ the partial pressure of the air in the cylinder all the time is $p_0$ and balances the air pressure outside the cylinder. It means that only the gas from the cavity effectively performs the work.

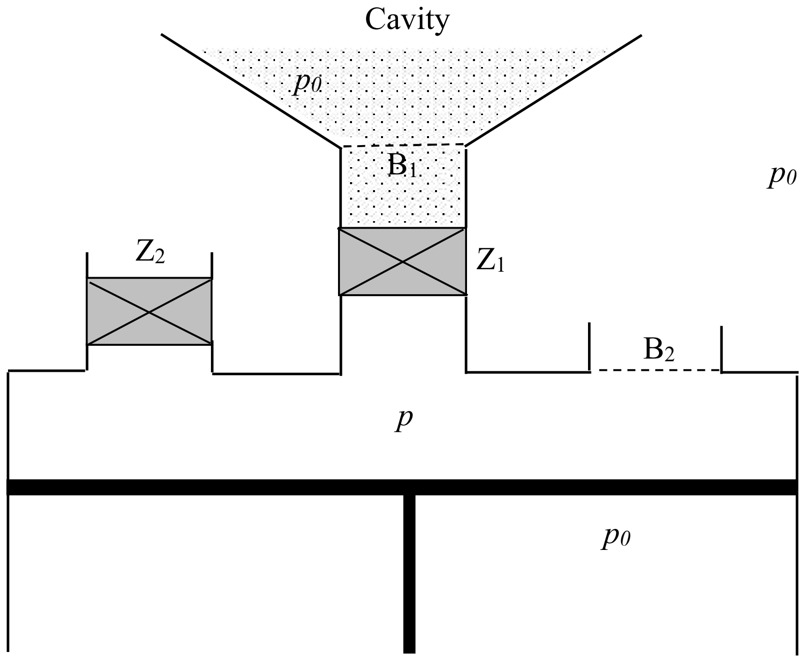


Fig. 6

Consider the problem of limits for the work that can be performed during isothermal expansion of an initial portion of the gas. Let us analyze the graph of the function $p_0 V_0 / V$ versus $V$ shown in Fig. 7.

It is obvious that the amount of work performed by the gas during isothermal expansion from $V_0$ to $V_k$ is represented by the area under the curve (shown in the graph) from $V_0$ to $V_k$. Of course, the work is proportional to $V_0$. We shall prove that for large enough $V_k$ the work can be arbitrarily large.

Consider $V = V_0, 2V_0, 4V_0, 8V_0, 16V_0, ...$ It is clear that the rectangles I, II, III, … (see Fig. 7) have the same area and that one may draw arbitrarily large number of such rectangles under the considered curve. It means that during isothermal expansion of a given portion of the gas we may obtain arbitrarily large work (at the cost of the heat taken from sthe urrounding) – it is enough to take $V_k$ large enough.

After reaching $V_k$ we open the valve $Z_2$ and move the piston to its initial position without performing any work. The cycle can be repeated as many times as we want.

In the above considerations we focused our attention on the work obtained during one cycle only. We entirely neglected dynamics of the process, while each cycle lasts some time. One may think that - in principle - the length of the cycle increases very rapidly with the effective work we obtain. This would limit the power of the device we consider.

Take, however, into account that, by proper choice of various parameters of the device, the time taken by one cycle can be made small and the initial volume of the gas $V_0$ can be made arbitrarily large (we consider only theoretical possibilities – we neglect practical difficulties entirely). E.g. by taking large size of the membrane $B_1$ and large size of the piston we may minimize the time of taking the initial portion of the gas $V_0$ from the cavity and make this portion very great.

In our analysis we neglected all losses, friction, etc. One should remark that there are no theoretical limits for them. These losses, friction etc. can be made negligibly small.

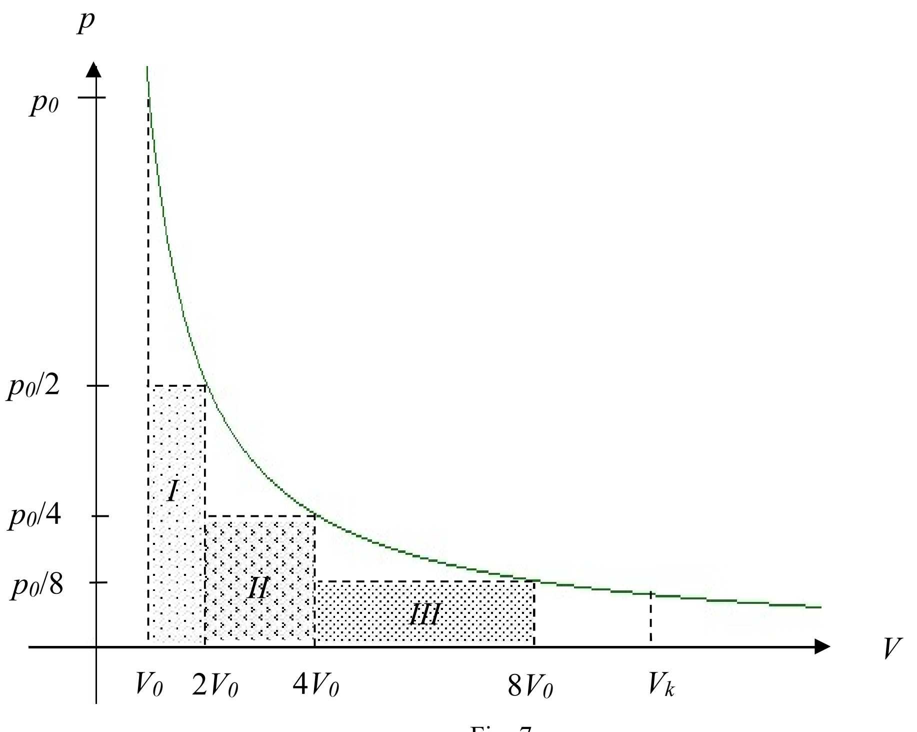


Fig. 7

The device we analyzed is very interesting: it produces work at cost of heat taken from surrounding without any difference in temperatures. Does this contradict the second law of thermodynamics? No! It is true that there is no temperature difference in the system, but the work of the device makes irreversible changes in the system (mixing of the gas from the cavity and the air).

The solutions were marked according to the following scheme (draft):

| | | |
|---|---|---|
| 1. | Model of an engine and its description | up to 4 points |
| 2. | Proof that there is no theoretical limit for power | up to 4 points |
| 3. | Remark on II law of thermodynamics | up to 2 points |