Solution of the experimental problem

a) Determination of the density of the material

The average density of the two bodies was chosen so that the bodies float on the water. Using the mass of the liquid crowded out it is determined the mass of the first body (the homogenous body):

$$m = m_a = V_a \rho_a = S_a H \rho_a \qquad (1)$$

where S_a is the area of the base immersed in water, H the length of the cylinder and ρ_a is the density of water.

The mass of the cylinder is:

$$m = V \cdot \rho = \pi R^2 H \rho \qquad (2)$$

It results the density of the body:

$$\rho = \rho_a \frac{S_a}{\pi R^2} \qquad (3)$$

To calculate the area S_a it is measured the distance h above the water surface (fig. 5.1). Area is composed by the area of the triangle OAB plus the area of the circular sector with the angle $2\pi - 2\theta$.

The triangle area:

$$\frac{1}{2} \cdot 2\sqrt{R^2 - (R-h)} \cdot (R-h) = (R-h)\sqrt{h(2R-h)} \qquad (4)$$

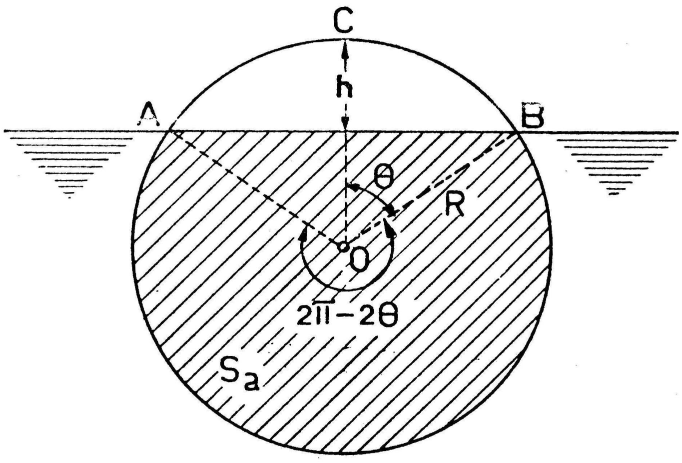

Fig. 5.1

The circular sector area is:

$$\frac{2(\pi - \theta)}{2\pi} \pi R^2 = R^2 \left(\pi - \arccos \frac{R-h}{R} \right) \qquad (5)$$

The immersed area is:

$$S_a = (R-h)\sqrt{h(2R-h)} + R^2 \left(\pi - \arccos \frac{R-h}{R} \right) \qquad (6)$$

where R and h are measured by the graduated rule.

b) The radius of the cylindrical cavity

The second body (with cavity) is dislocating a water mass:

$$m^{'} = m_a^{'} = S_a^{'} H \rho_a \qquad (7)$$

where S_a' is area immersed in water.

The mass of the body having the cavity inside is:

$$m^{'} = (V - v)\rho = \pi\left(R^2 - r^2\right)H\rho \qquad (8)$$

The cavity radius is:

$$r = \sqrt{R^2 - \frac{\rho_a}{\pi\rho} \cdot S_a^{'}} \qquad (9)$$

$S_a^{'}$ is determined like S_a.

c) The distance between the cylinder's axis and the cavity axis

We put the second body on the horizontal table (or let it to float in water) and we trace the vertical symmetry axis AB (fig. 5.2).

Using the rule we make an inclined plane. We put the body on this plane and we determine the maximum angle of the inclined plane for the situation the body remains in rest (the body doesn't roll). Taking in account that the weight centre is located on the axis AB on the left side of the cylinder axis (point G in fig. 5.2) and that at equilibrium the weight centre is on the same vertical with the contact point between the cylinder and the inclined plane, we obtain the situation corresponding to the maximum angle of the inclined plane (the diameter AB is horizontal).

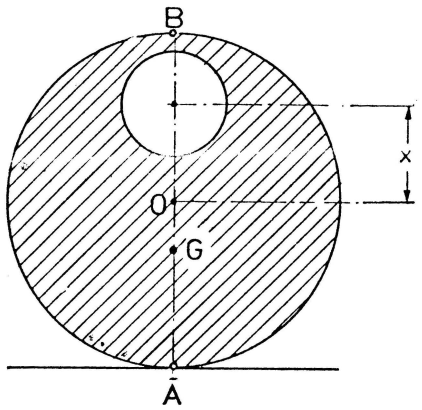

Fig. 5.2

The distance OG is calculated from the equilibrium condition:

$$m^{'} \cdot OG = m_c \cdot x \text{ , (} m_c \text{ = the mass dislocated by the cavity)} \qquad (10)$$

$$OG = R\sin\alpha \qquad (11)$$

$$x = OG \cdot \frac{m^{'}}{m_c} = R \cdot \sin\alpha \cdot \frac{R^2 - r^2}{r^2} \qquad (12)$$

d) At every measurement it must be estimated the reading error. Taking in account the expressions for ρ, r and x it is evaluated the maximum error for the determination of these measures.