Solution problem 4

a) From the Fermat principle it results that the time the light arrives from P_1 to P_2 is not dependent of the way, in gauss approximation (P_1 and P_2 are conjugated points).

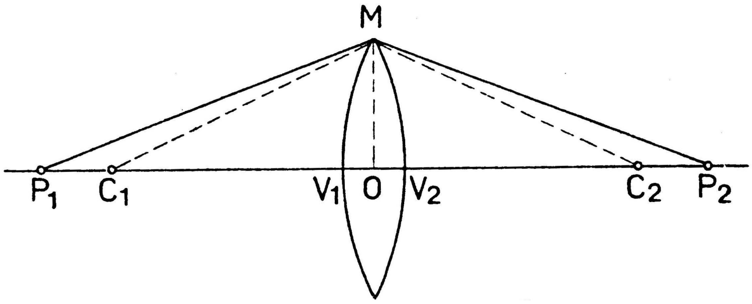

Fig. 4.2

T_1 is the time the light roams the optical way $P_1V_1OV_2P_2$ (fig. 4.2):

$T_1 = \frac{P_1M}{v_1} + \frac{P_2M}{v_2}$, where $P_1M = \sqrt{P_1O^2 + MO^2} \approx P_1O + \frac{h^2}{2P_1O}$, and $P_2M \approx P_2O + \frac{h^2}{2P_2O}$

because $h = OM$ is much more smaller than P_1O or P_2O.

$$T_1 = \frac{P_1O}{v_1} + \frac{P_2O}{v_2} + \frac{h^2}{2} \cdot \left(\frac{1}{v_1P_1O} + \frac{1}{v_2P_2O} \right); \; T_2 = \frac{P_1V_1}{v_1} + \frac{V_2P_2}{v_2} + \frac{V_1V_2}{v} \quad (1)$$

$$V_1V_2 \cong \frac{h^2}{2} \cdot \left(\frac{1}{R_1} + \frac{1}{R_2} \right) \quad (2)$$

From condition $T_1 = T_2$, it results:

$$\frac{1}{v_1P_1O} + \frac{1}{v_2P_2O} = \frac{1}{v}\left(\frac{1}{R_1} + \frac{1}{R_2} \right) - \frac{1}{v_1R_1} - \frac{1}{v_2R_2} \quad (3)$$

Taking in account the relation $v = \frac{c}{n}$, and using $P_1O = s_1, OP_2 = s_2$, the relation (3) can be written:

$$\frac{n_1}{s_1} + \frac{n_2}{s_2} = n_o\left(\frac{1}{R_1} + \frac{1}{R_2} \right) - \frac{1}{v_1R_1} - \frac{1}{v_2R_2} \quad (4)$$

If the point P_1 is at infinite, s_2 becomes the focal distance; the same for P_2.

$$\frac{1}{f_2} = \frac{1}{n_2} \cdot \left(\frac{n_o - n_1}{R_1} + \frac{n_o - n_2}{R_2} \right); \quad \frac{1}{f_1} = \frac{1}{n_1} \cdot \left(\frac{n_o - n_1}{R_1} + \frac{n_o - n_2}{R_2} \right) \quad (5)$$

From the equations (30 and (4) it results:

$$\frac{f_1}{s_1} + \frac{f_2}{s_2} = 1 \quad (6)$$

The lens is plane-convex (fig. 4.3) and its focal distances are:

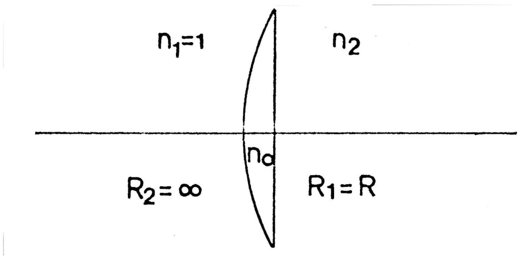

Fig. 4.3

$$f_1 = \frac{n_1 R}{n_o - n_1} = \frac{R}{n_o - 1} \quad ; \quad f_2 = \frac{n_2 R}{n_o - n_1} = \frac{n_2 R}{n_0 - 1} \qquad (7)$$

b) In the case of Billet lenses, S_1 and S_2 are the real images of the object S and can be considered like coherent light sources (fig. 4.4).

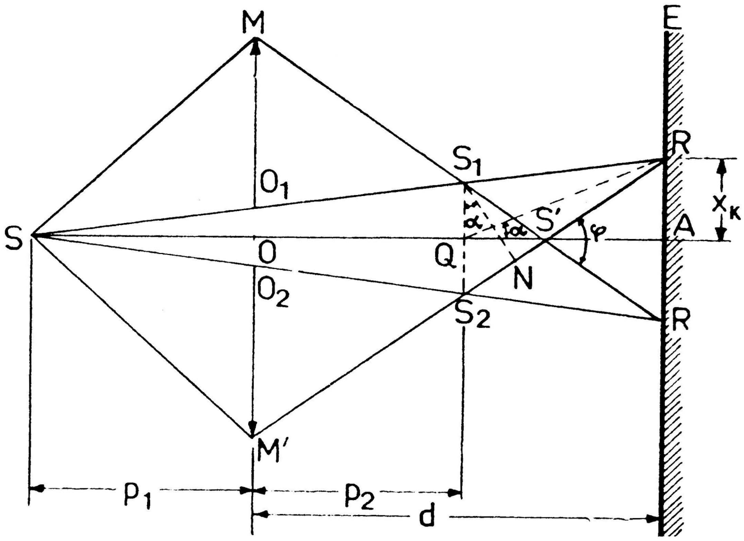

Fig. 4.4

$O_1O_2 = \Delta$ is much more smaller than r:

$$OM = \Delta + r \approx r,\ SO \approx SO_1 \approx SO_2 = p_1,\ S_1O_1 = S_2O_2 \approx S'O = p_2,\ S_1S_2 = \Delta \cdot \left(1 + \frac{p_1}{p_2}\right)$$

We calculate the width of the interference field RR' (fig. 4.4).

$$RR' = 2 \cdot RA = 2 \cdot S'A \cdot tg\frac{\varphi}{2},\ S'A \cong d - p_2,\ tg\frac{\varphi}{2} = \frac{r}{p_2},\quad RR' = 2(d - p_2) \cdot \frac{r}{p_2}$$

Maximum interference condition is:

$$S_2N = k \cdot \lambda$$

The fringe of k order is located at a distance x_k from A:

$$x_k = k \cdot \frac{\lambda(d - p_2)}{\Delta\left(1 + \frac{p_2}{p_1}\right)} \qquad (8)$$

The expression of the inter-fringes distance is:

$$i = \frac{\lambda(d - p_2)}{\Delta\left(1 + \frac{p}{p_1}\right)} \qquad (9)$$

The number of observed fringes on the screen is:

$$N = \frac{RR^{'}}{i} = 2r\Delta \cdot \frac{1 + \frac{p_2}{p_1}}{\lambda p_2} \qquad (10)$$

p_2 can be expressed from the lenses' formula:

$$p_2 = \frac{p_1 f}{p_1 - f}$$