Solution Problem 1

The inertia moments of the three cylinders are:

$$I_1 = \frac{1}{2}\rho_1\pi\left(R^4 - r^4\right)h, \quad I_2 = \frac{1}{2}\rho_2\pi R^4 h = \frac{1}{2}mR^2 \quad , \quad I3 = \frac{1}{2}\rho_2\pi\left(R^4 - r^4\right)h, \qquad (1)$$

Because the three cylinders have the same mass :

$$m = \rho_1\pi\left(R^2 - r^2\right)h = \rho_2\pi R^2 h \qquad (2)$$

it results:

$$r^2 = R^2\left(1 - \frac{\rho_2}{\rho_1}\right) = R^2\left(1 - \frac{1}{n}\right), n = \frac{\rho_1}{\rho_2} \qquad (3)$$

The inertia moments can be written:

$$I_1 = I_2\left(2 - \frac{1}{n}\right)\rangle I_2 \ , \quad I_3 = I_2\left(2 - \frac{1}{n}\right)\cdot\frac{1}{n} = \frac{I_1}{n} \qquad (4)$$

In the expression of the inertia momentum I_3 the sum of the two factors is constant:

$$\left(2 - \frac{1}{n}\right) + \frac{1}{n} = 2$$

independent of n, so that their products are maximum when these factors are equal: $2 - \frac{1}{n} = \frac{1}{n}$; it results n = 1, and the products $\left(2 - \frac{1}{n}\right)\cdot\frac{1}{n} = 1$. In fact n > 1, so that the products is les than 1. It results:

$$I_1 > I_2 > I_3 \qquad (5)$$

For a cylinder rolling over freely on the inclined plane (fig. 1.1) we can write the equations:

$$mg\sin\alpha - F_f = ma \qquad (6)$$

$$N - mg\cos\alpha = 0$$

$$F_f R = I\varepsilon \qquad (7)$$

where ε is the angular acceleration. If the cylinder doesn't slide we have the condition:

$$a = \varepsilon R \qquad (8)$$

Solving the equation system (6-8) we find:

$$a = \frac{g\sin\alpha}{1+\frac{I}{mR^2}} \quad , \quad F_f = \frac{mg\sin\alpha}{1+\frac{mR^2}{I}} \tag{9}$$

The condition of non-sliding is:

$$F_f < \mu N = \mu mg\sin\alpha$$

$$\mathrm{tg}\alpha \; < \; \mu\left(1+\frac{mR^2}{I_1}\right) \tag{10}$$

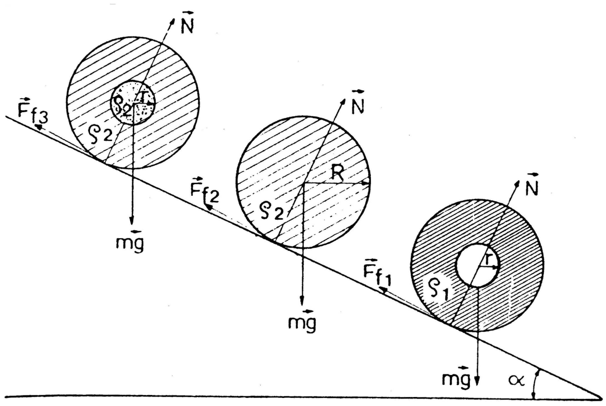

Fig. 1.1

In the case of the cylinders from this problem, the condition necessary so that none of them slides is obtained for maximum I:

$$tg\alpha \langle \mu\left(1+\frac{mR^2}{I_1}\right) = \mu\frac{4n-1}{2n-1} \tag{11}$$

The accelerations of the cylinders are:

$$a_1 = \frac{2g\sin\alpha}{3+(1-\frac{1}{n})} \quad , \quad a_2 = \frac{2g\sin\alpha}{3} \quad , \quad a_3 = \frac{2g\sin\alpha}{3-(1-\frac{1}{n})^2}. \tag{12}$$

The relation between accelerations:

$$a_1 < a_2 < a_3 \tag{13}$$

In the case than all the three cylinders slide:

$$F_f = \mu N = \mu mg\cos\alpha \tag{14}$$

and from (7) results:

$$\varepsilon = \frac{R}{I}\mu mg\cos\alpha \qquad (15)$$

for the cylinders of the problem:

$$\varepsilon_1 : \varepsilon_2 : \varepsilon_3 = \frac{1}{I_1} : \frac{1}{I_2} : \frac{1}{I_3} = 1 : \left(1 - \frac{1}{n}\right) : n$$

$$\varepsilon_1 < \varepsilon_2 < \varepsilon_3 \qquad (16)$$

In the case that one of the cylinders is sliding:

$$mg\sin\alpha - F_f = ma\,, \quad F_f = \mu mg\cos\alpha\,, \quad (17)$$

$$a = g(\sin\alpha - \mu\cos\alpha) \qquad (18)$$

Let $\vec{F}$ be the total force acting on the liquid mass m_l inside the cylinder (fig.1.2), we can write:

$$F_x + m_l g\sin\alpha = m_l a = m_l g(\sin\alpha - \mu\cos\alpha), \;\; F_y - m_l g\cos\alpha = 0 \quad (19)$$

$$F = \sqrt{F_x^2 + F_y^2} = m_l g\cos\alpha \cdot \sqrt{1+\mu^2} = m_l g\frac{\cos\alpha}{\cos\phi} \qquad (20)$$

where ϕ is the friction angle $(tg\phi = \mu)$.

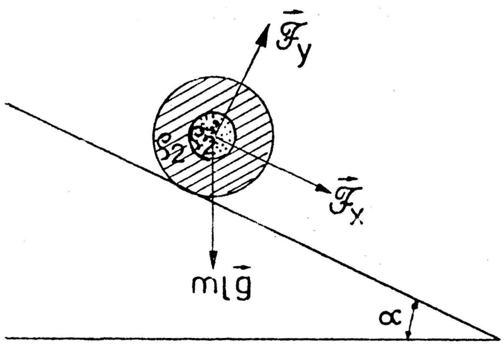

Fig. 1.2