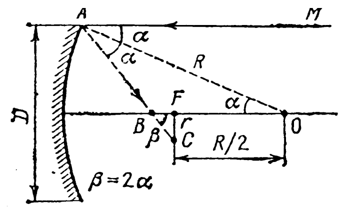
Problem 4.



As known, rays parallel to the main optical axis of a spherical mirror, passing at little distances from it after having been reflected, join at the main focus of the mirror *F* which is at the distance *R*/2 from the centre *O* of the spherical surface. Let us consider now the movement of the ray reflected near the edge of the spherical mirror of large diameter *D* (Fig. 6). The angle of incidence α of the ray onto the surface is equal to the angle of reflection. That is why the angle *OAB* within the triangle, formed by the radius *OA* of the sphere, traced to the incidence point of the ray by the reflected ray *AB* and an intercept BO of the main optical axis, is equal to α. The angles BOA and MAO are equal, that is the angle BOA is equal to α.

Thus, the triangle AOB is isosceles with its side AB being equal to the side BO. Since the sum of the lengths of its two other sides exceeds the length of its third side, *AB+BO>OA=R, hence BO>R*/2. This means that a ray parallel to the main optical axis of the spherical mirror and passing not too close to it, after having been reflected, crosses the main optical axis at the point *B* lying between the focus *F* and the mirror. The focal surface is crossed by this ray at the point *C* which is at a certain distance *CF* = r from the main focus.

Thus, when reflecting a parallel beam of rays by a spherical mirror finite in size it does not join at the focus of the mirror but forms a beam with radius *r* on the focal plane.

From Δ *BFC* we can write :

$$r = BF\,\mathrm{tg}\,\beta = BF\,\mathrm{tg}\,2\alpha\,,$$

where α is the maximum angle of incidence of the extreme ray onto the mirror, while $\sin\alpha = D/2R$:

$$BF = BO - OF = \frac{R}{2\cos\alpha} - \frac{R}{2} = \frac{R}{2}\frac{1-\cos\alpha}{\cos\alpha}.$$

Thus, $r = \dfrac{R}{2}\dfrac{1-\cos\alpha}{\cos\alpha}\dfrac{\sin 2\alpha}{\cos 2\alpha}$. Let us express the values of $\cos\alpha$, $\sin 2\alpha$, $\cos 2\alpha$ via $\sin\alpha$ taking into account the small value of the angle α:

$$\cos\alpha = \sqrt{1-\sin^2\alpha} \approx 1 - \frac{\sin^2\alpha}{2},$$

$$\sin 2\alpha = 2\sin\alpha\cos\alpha\,,$$

$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha = 1 - 2\sin^2\alpha\,.$$

Then

$$r = \frac{R}{2}\frac{\sin^3\alpha}{1-2\sin^2\alpha} \approx \frac{R}{2}\sin^3\alpha \approx \frac{D^3}{16R^2}.$$

Substituting numerical data we will obtain: $r \approx 1.95 \cdot 10^{-3}$ m $\approx$ 2mm .

From the expression $D = \sqrt[3]{16R^2 r}$ one can see that if the radius of the receiver is decreased 8 times the transversal diameter *D*' of the mirror, from which the light comes to the receiver, will be decreased 2 times and thus the "effective" area of the mirror will be decreased 4 times.

The radiation flux Φ reflected by the mirror and received by the receiver will also be decreased twice since Φ ~ *S*.

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