Problem 3.

Having no charge on the ball the sphere has the potential

$$\varphi_{0s} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R} = 450\text{V}.$$

When connected with the Earth the ball inside the sphere has the potential equal to zero so there is an electric field between the ball and the sphere. This field moves a certain charge q from the Earth to the ball. Charge Q`, uniformly distributed on the sphere, doesn't create any field inside thus the electric field inside the sphere is defined by the ball's charge q. The potential difference between the balls and the sphere is equal

$$\Delta\varphi = \varphi_b - \varphi_s = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r} - \frac{q}{R}\right), \qquad (1)$$

Outside the sphere the field is the same as in the case when all the charges were placed in its center. When the ball was connected with the Earth the potential of the sphere φ_s is equal

$$\varphi_s = \frac{1}{4\pi\varepsilon_0}\frac{q+Q}{R}. \qquad (2)$$

Then the potential of the ball

$$\varphi_b = \varphi_s + \Delta\varphi = \frac{1}{4\pi\varepsilon_0}\left(\frac{q+Q}{R} + \frac{q}{r} - \frac{q}{R}\right) = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{R} + \frac{q}{r}\right) = 0 \qquad (3)$$

Which leads to

$$q = -Q\frac{r}{R}. \qquad (4)$$

Substituting (4) into (2) we obtain for potential of the sphere to be found:

$$\varphi_s = \frac{1}{4\pi\varepsilon_0}\frac{Q - Q\frac{r}{R}}{R} = \frac{1}{4\pi\varepsilon_0}\frac{Q(R-r)}{R^2} = 225\text{V}.$$

The electric capacity of whole system of conductors is

$$C = \frac{Q}{\varphi_s} = \frac{4\pi\varepsilon_0 R^2}{R-r} = 4.4\cdot 10^{-11}\,\text{F} = 44\text{pF}$$

The equivalent electric scheme consists of two parallel capacitors: 1) a spherical one with charges $+q$ and $-q$ at the plates and 2) a capacitor "sphere – Earth" with charges $+(Q-q)$ and $-(Q-q)$ at the plates (Fig.5).

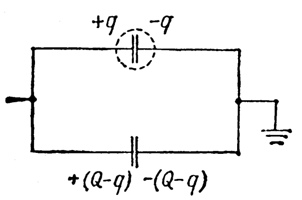

Fig. 5